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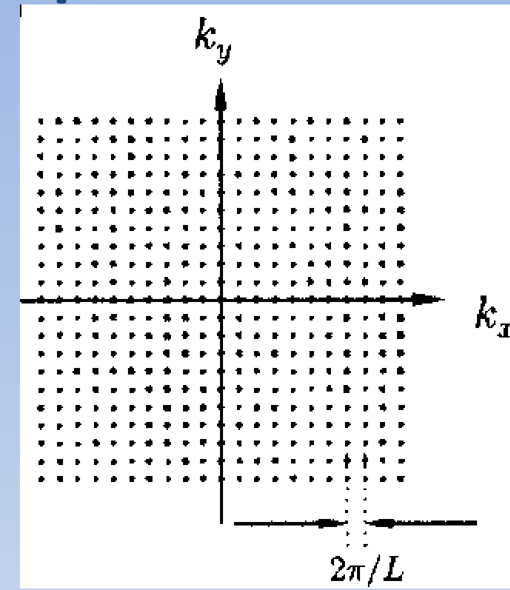
Физика магнетизма и рассеяние поляризованных и неполяризованных нейтронов

Лекция 10. Магнетизм металлов. Парамагнетизм Паули. Диамагнетизм Ландау.

Модель свободных электронов

$$g(k)dk = \frac{2}{V(2\pi/L)^3}4\pi k^2 dk = \frac{k^2 dk}{\pi^2}$$



$$E = \frac{\hbar^2 k^2}{2m}$$

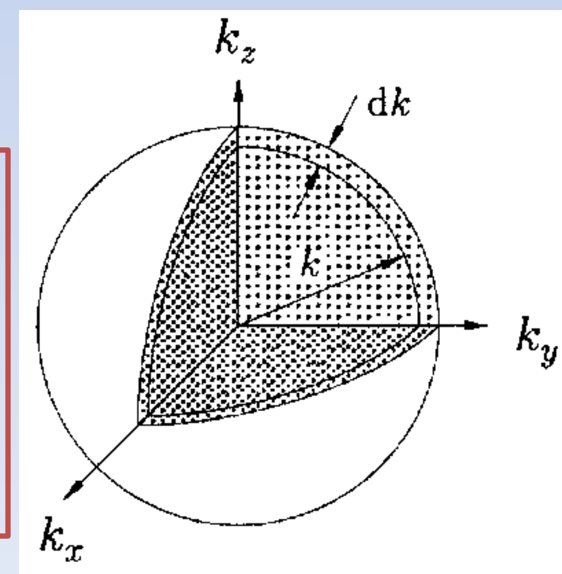
$$dE = \frac{\hbar^2 k}{m}dk = \sqrt{\frac{2m}{\hbar^2}E}\,\frac{\hbar^2}{m}dk = \sqrt{E}\sqrt{\frac{2}{m}}\hbar dk$$

$$g(E)dE = \frac{1}{\pi^2}\frac{2mE}{\hbar^2}\frac{dE}{\hbar\sqrt{E}}\sqrt{\frac{m}{2}} = \frac{\sqrt{2}m^{3/2}}{\pi^2\hbar^3}\sqrt{E}dE$$

$$n = \int_0^{k_F} g(k)dk = \frac{k_F^3}{3\pi^2}$$

$$k_F^3 = 3\pi^2 n, \qquad E_F = \frac{\hbar^2 k_F^2}{2m}$$

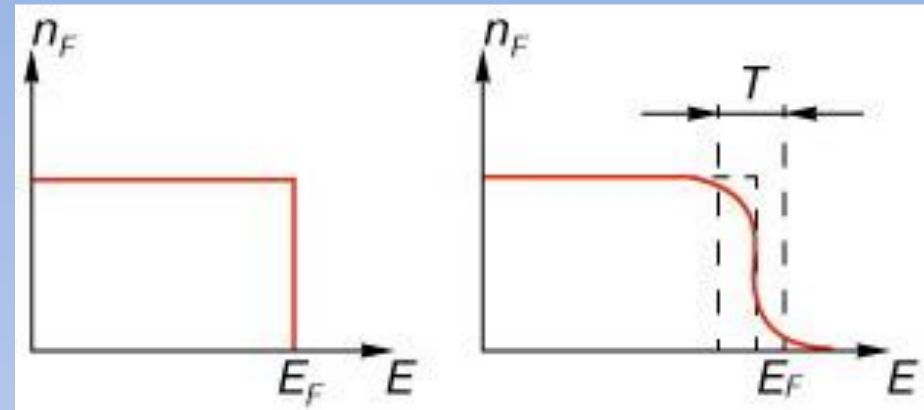
$$g(E_F) = \frac{mk_F}{\pi^2\hbar^2} = \frac{3}{2}\frac{n}{E_F}$$



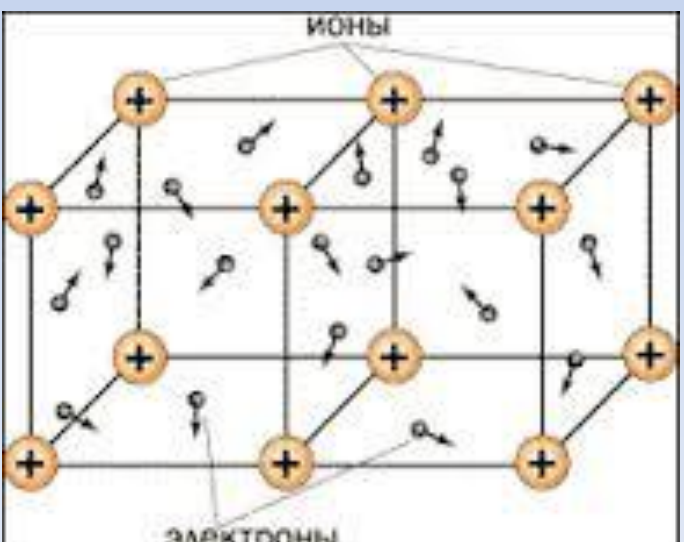
$$n_F(E)=\frac{1}{e^{(E-\mu)/k_BT}+1}$$

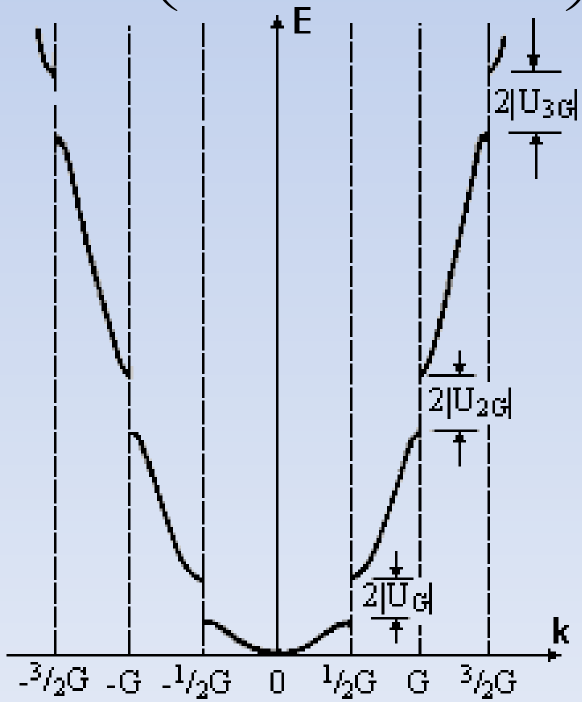
$$\int_0^{E_F} g(E)\,n_F(E)\,dE=n$$

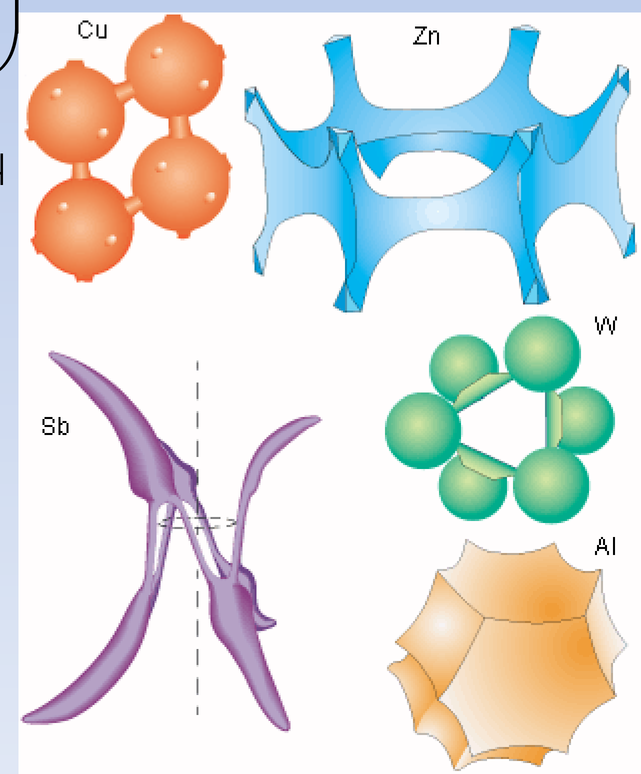
$$T=0: \qquad \mu=E_F$$



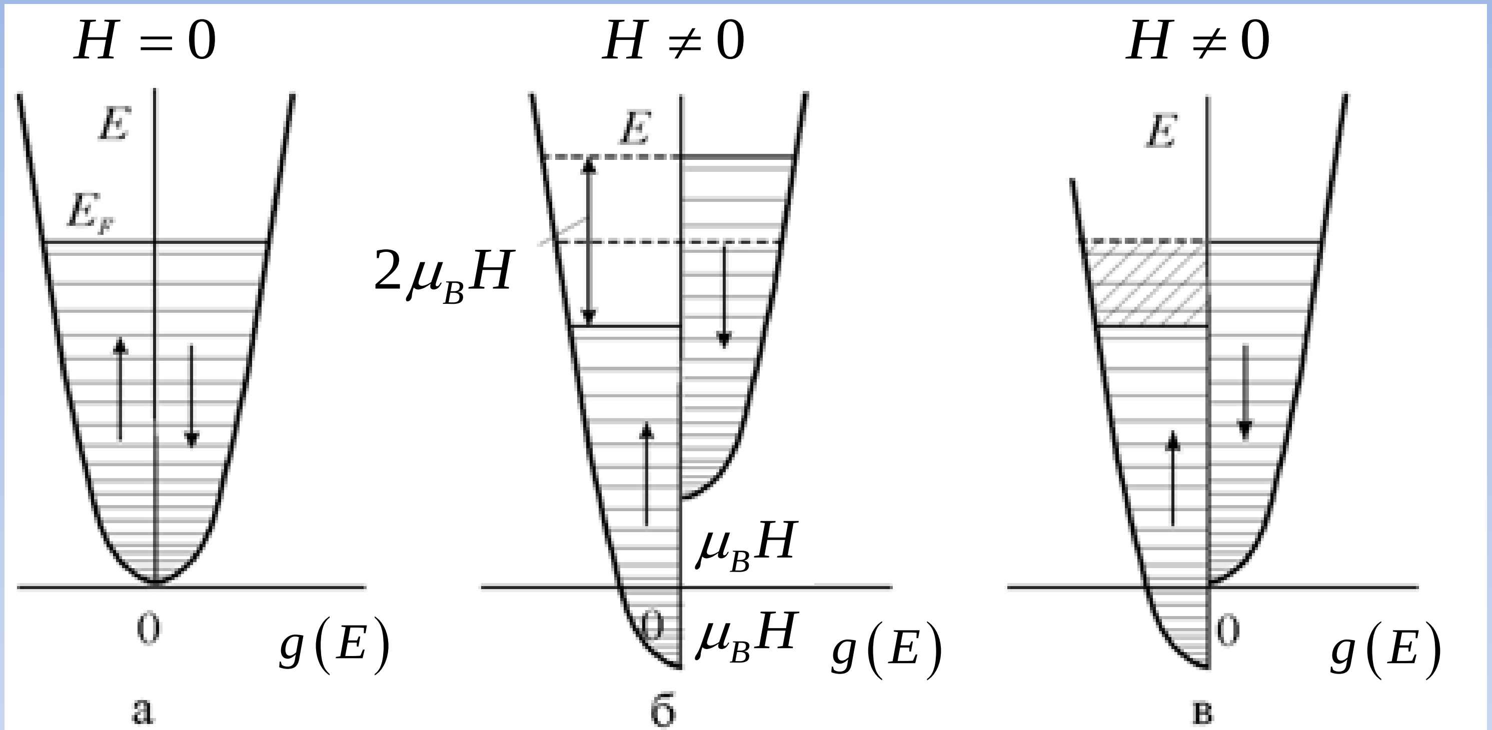
$$T \ll E_F \sim 10^5\ \text{K}: \qquad \mu\approx E_F\left(1-\frac{\pi^2}{12}\left(\frac{k_BT}{E_F}\right)^2\right)$$







Парамагнетизм Паули



$$\Delta n_{\uparrow}=\frac{1}{2}g\left(E_F\right)\mu_B H \qquad \Delta n_{\downarrow}=-\frac{1}{2}g\left(E_F\right)\mu_B H$$

$$M=\mu_B\left(\Delta n_{\uparrow}-\Delta n_{\downarrow}\right)=g\left(E_F\right)\mu_B^2 H$$

$$\chi_P=g\left(E_F\right)\mu_B^2=\frac{mk_F}{\pi^2\hbar^2}\mu_B^2=\frac{3}{2}\frac{n\mu_B^2}{E_F}$$

$$n_\uparrow = \frac{1}{2}\int_{-\infty}^{\infty} g\left(E+\mu_B H\right) n_F\left(E\right) dE \qquad n_\downarrow = \frac{1}{2}\int_{-\infty}^{\infty} g\left(E-\mu_B H\right) n_F\left(E\right) dE$$

$$M = \mu_B\left(n_\uparrow - n_\downarrow\right) = \mu_B^2 H \int_{-\infty}^{\infty} \frac{dg\left(E\right)}{dE} n_F\left(E\right) dE$$

$$= \mu_B^2 H\left(\left[g\left(E\right) n_F\left(E\right)\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{dn_F\left(E\right)}{dE} g\left(E\right) dE\right)$$

$$= -\mu_B^2 H \int_{-\infty}^{\infty} \frac{dn_F\left(E\right)}{dE} g\left(E\right) dE \qquad -\frac{dn_F\left(E\right)}{dE} = \begin{cases} \delta\left(E-E_F\right), & T \ll E_F \\ \dfrac{n_F\left(E\right)}{k_B T}, & T > E_F \end{cases}$$

$$M = \begin{cases} \mu_B^2 g\left(E_F\right) H = \dfrac{3}{2}\dfrac{n\mu_B^2}{E_F} H, & T \ll E_F \\ \dfrac{\mu_B^2 H}{k_B T}\displaystyle\int_0^{\infty} n_F\left(E\right) g\left(E\right) dE = \dfrac{n\mu_B^2}{k_B T} H, & T > E_F \end{cases} \quad \Rightarrow \quad \chi = \frac{n\mu_B^2}{k_B T}$$

характерно для допированных полупроводников с малыми n и $E_F \propto n^{2/3}$

Зонные магнетики (itinerant magnets)

$$\Delta E_{kin}=\left(\frac{1}{2}g\left(E_F\right)\delta E\right)\delta E=g\left(E_F\right)\frac{\delta E^2}{2}>0$$

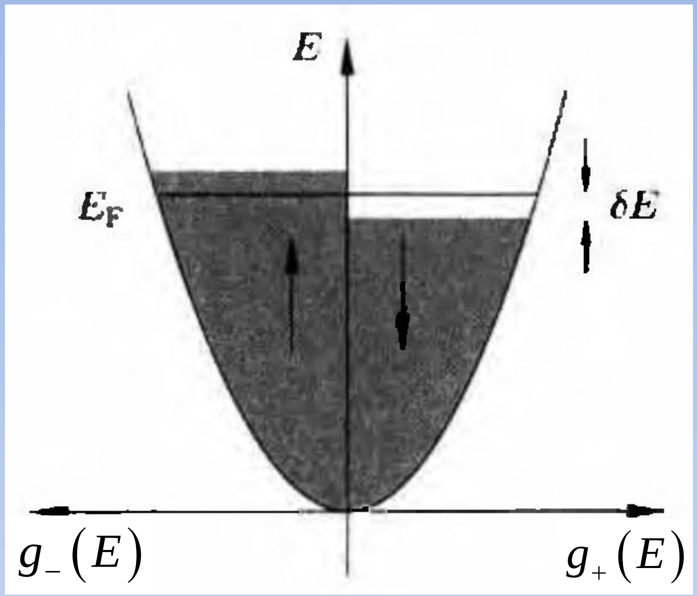
$$M=\mu_B\left(n_\uparrow-n_\downarrow\right)=\mu_B g\left(E_F\right)\delta E$$

$$H_{mf}=\lambda M$$

$$\Delta E_{mf}=-\lambda\int_0^M M'dM'=-\frac{\lambda}{2}M^2$$

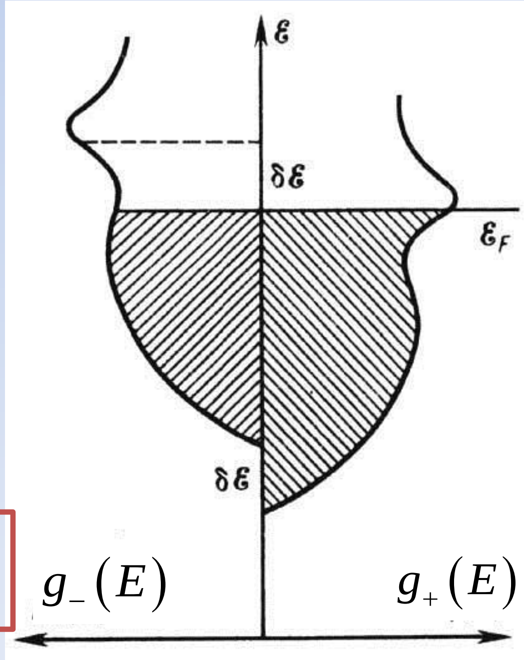
$$=-\frac{\lambda}{2}\left(\mu_B g\left(E_F\right)\delta E\right)^2=-\lambda\mu_B^2 g\left(E_F\right)\left(g\left(E_F\right)\frac{\delta E^2}{2}\right)$$

$$\Delta E=g\left(E_F\right)\frac{\delta E^2}{2}\left(1-Ug\left(E_F\right)\right)$$



$$U=\lambda\mu_B^2$$

критерий ферромагнетизма Стонера $Ug\left(E_F\right)>1$



«Стонеровское» усиление восприимчивости

$$\Delta E = g\left(E_F\right)\frac{\delta E^2}{2}\left(1 - Ug\left(E_F\right)\right) - MH = \frac{M^2}{2\mu_B^2 g\left(E_F\right)}\left(1 - Ug\left(E_F\right)\right) - MH$$

$$\frac{d\Delta E}{dM} = \frac{M}{\mu_B^2 g\left(E_F\right)}\left(1 - Ug\left(E_F\right)\right) - H = 0$$

$$\boxed{\chi = \frac{\mu_B^2 g\left(E_F\right)}{1 - Ug\left(E_F\right)} = \frac{\chi_P}{1 - Ug\left(E_F\right)}}$$

другой способ:

$$H_{mf} = \lambda M$$

$$M = \chi_P H_{eff} = \chi_P\left(H + H_{mf}\right) = \chi_P\left(H + \lambda M\right)$$

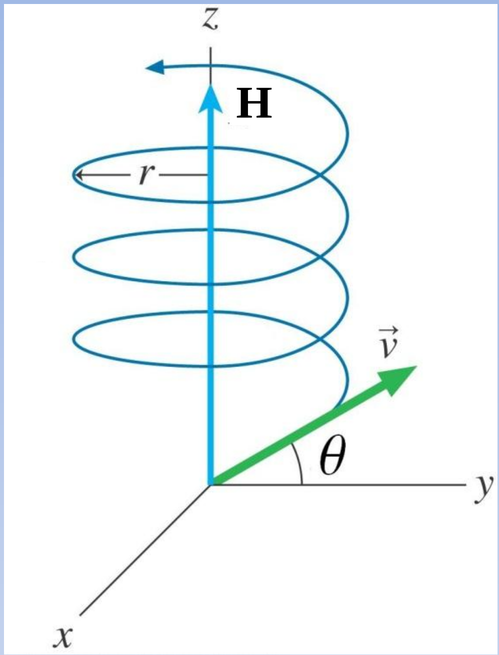
$$M = \frac{\chi_P}{1 - \lambda\chi_P} H$$

Пример: Pd, Pt

Электроны в магнитном поле

$$m\ddot{\mathbf{r}} = \mathbf{F} = -\frac{e}{c}\left[\dot{\mathbf{r}} \times \mathbf{H}\right]$$

$$\omega_c = \frac{eH}{mc}$$



$$\ddot{x} = -\omega_c \dot{y} \qquad \ddot{y} = \omega_c \dot{x} \qquad \ddot{z} = 0$$

$$x = r\cos\left(\omega_c t + \varphi\right) + x_0$$

$$y = r\sin\left(\omega_c t + \varphi\right) + y_0 \qquad z = \mathrm{v}_{z0} t + z_0$$

$$y_0 = y + \frac{\mathrm{v}_x}{\omega_c} \qquad x_0 = x - \frac{\mathrm{v}_y}{\omega_c}$$

$$\mathbf{H} = \left[\nabla \times \mathbf{A}\right] \qquad \mathbf{A} = \left(-Hy, 0, 0\right)$$

$$\mathcal{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{q}{c}\mathbf{A}\right)^2 = \frac{1}{2m}\left(\hat{p}_x - \frac{e}{c}Hy\right)^2 + \frac{1}{2m}\hat{p}_y^2 + \frac{1}{2m}\hat{p}_z^2$$

$$\psi = \chi\left(y\right)e^{i\left(p_x x + p_z z\right)/\hbar}$$

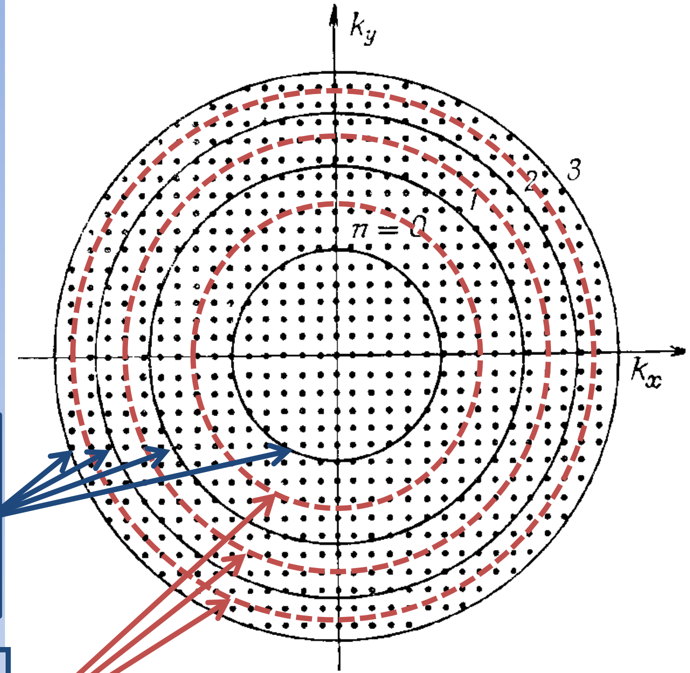
$$\hat{y}_0 = \frac{\hat{p}_x}{m\omega_c} = \frac{\hat{\mathrm{v}}_x}{\omega_c} + \hat{y}$$

$$\chi'' + \frac{2m}{\hbar^2}\left[\left(E - \frac{p_z^2}{2m}\right) - \frac{m}{2}\omega_c^2\left(y - y_0\right)^2\right]\chi = 0$$

$$\hat{x}_0 = \hat{x} - \frac{\hat{\mathrm{v}}_y}{\omega_c}$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{1}{2m}p_z^2$$

$$a = \sqrt{\frac{\hbar}{m\omega_c}}$$



$$\chi_n(y) = \frac{1}{\pi^{1/4}\sqrt{a2^n n!}} e^{-(y-y_0)^2/2a^2} H_n\left(\frac{y-y_0}{a}\right)$$

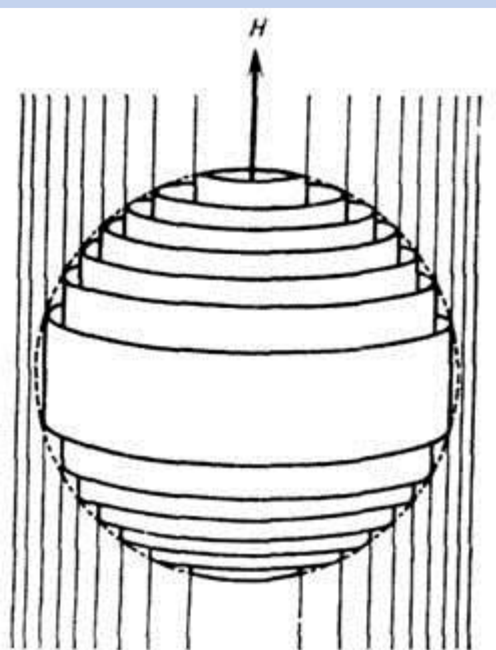
$$0 < y_0 < L_y$$

$$0 < p_x < m\omega_c L_y$$

$$P = m\omega_c L_y \frac{L_x}{2\pi\hbar}$$

$$\frac{\hbar^2 k_n^2}{2m} = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

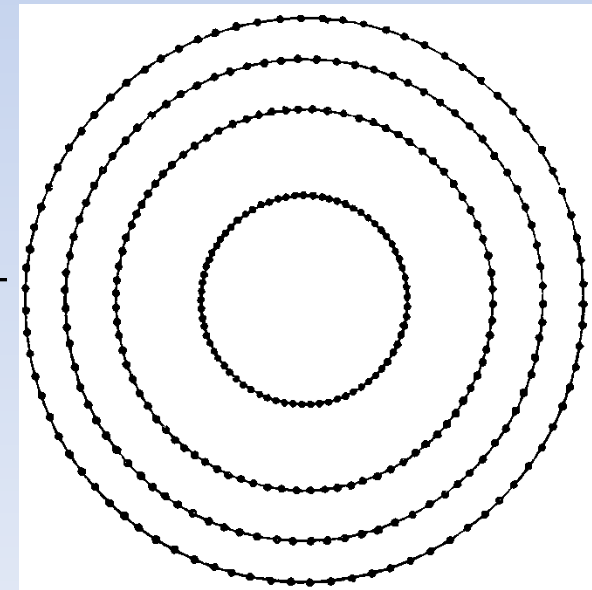
$$\frac{\hbar^2 k_{n+1/2}^2}{2m} = (n+1)\hbar\omega_c$$



$$\frac{L_x L_y}{(2\pi)^2}\left(\pi k_{n+3/2}^2 - \pi k_{n+1/2}^2\right)$$

$$= \frac{L_x L_y}{(2\pi)^2}\frac{2\pi m\omega_c}{\hbar} = m\omega_c \frac{L_x L_y}{2\pi\hbar}$$

$$= \frac{L_x L_y}{(2\pi)^2}\pi k_{1/2}^2$$



Диамагнетизм Ландау

$$\Omega = -T\sum_j \ln\left(1+e^{(\mu-E_j)/T}\right) \qquad \frac{\partial\Omega}{\partial\mu} = -N = \sum_j n_F\left(E_j\right)$$

$$g_n\left(E\right)dE = m\omega_c \frac{L_x L_y}{2\pi\hbar} 4\frac{L_z}{2\pi\hbar}\frac{d\left|p_z\right|}{dE}dE = 4m\omega_c \frac{V}{\left(2\pi\hbar\right)^2}\frac{d\left|p_z\right|}{dE}dE$$

$$\left|p_z\right| = \sqrt{2m\left(E-\left(n+\frac{1}{2}\right)\hbar\omega_c\right)}$$

$$\Omega = -T\frac{4m\omega_c V}{\left(2\pi\hbar\right)^2}\sum_{n=0}^{\infty}\int_{\left(n+\frac{1}{2}\right)\hbar\omega_c}^{\infty} dE\,\frac{d\left|p_z\right|}{dE}\ln\left(1+e^{(\mu-E)/T}\right)$$

$$= -\frac{4m\omega_c V}{\left(2\pi\hbar\right)^2}\sum_{n=0}^{\infty}\int_{\left(n+\frac{1}{2}\right)\hbar\omega_c}^{\infty} dE\,\frac{\left|p_z\right|}{e^{(E-\mu)/T}+1} \approx -\frac{4m\omega_c V}{\left(2\pi\hbar\right)^2}\sum_{n=0}^{\frac{\mu}{\hbar\omega_c}-\frac{1}{2}}\int_{\left(n+\frac{1}{2}\right)\hbar\omega_c}^{\mu} dE\left|p_z\right|$$

$$= -\frac{8m\omega_c V\sqrt{2m}}{3\left(2\pi\hbar\right)^2}\sum_{n=0}^{\frac{\mu}{\hbar\omega_c}-\frac{1}{2}}\left(\mu-\left(n+\frac{1}{2}\right)\hbar\omega_c\right)^{3/2}$$

$$\sum_{n=0}^{a} f(n) = \int_{-1/2}^{a+1/2} f(x)\,dx - \frac{1}{24}\left[f'\left(a+\frac{1}{2}\right) - f'\left(-\frac{1}{2}\right)\right]$$

$$\sum_{n=0}^{\frac{\mu}{\hbar\omega_c}-\frac{1}{2}} \left(\mu - \left(n+\frac{1}{2}\right)\hbar\omega_c\right)^{3/2} = \frac{2}{5\hbar\omega_c}\mu^{5/2} - \frac{\hbar\omega_c}{16}\sqrt{\mu}$$

$$\Omega = -\frac{16mV\sqrt{2m}}{15(2\pi\hbar)^2\hbar}\mu^{5/2} + \frac{m\hbar\omega_c^2 V\sqrt{2m}}{6(2\pi\hbar)^2}\sqrt{\mu}$$

$$M = -\frac{1}{V}\frac{\partial\Omega}{\partial H} = -\frac{m}{3\pi^2\hbar^2}\left(\frac{e\hbar}{2mc}\right)^2 k_F H$$

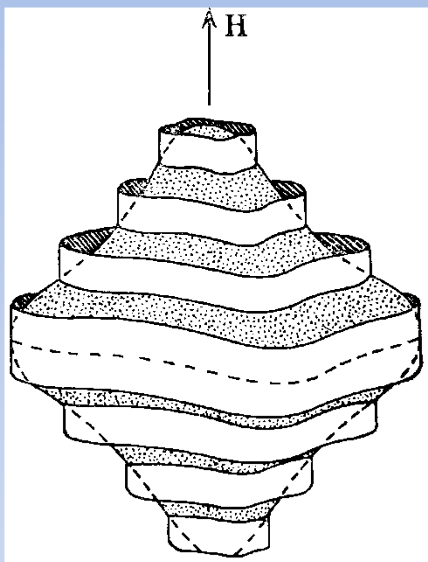
$$\chi_L = -\frac{1}{3}\frac{mk_F}{\pi^2\hbar^2}\mu_B^2 = -\frac{1}{3}\chi_P$$

$$\chi_L = -\frac{1}{3}\left(\frac{m}{m^*}\right)^2 \chi_P$$

Например, Bi диамагнитен: $m^* \sim 0.01m$

Осцилляции при изменении магнитного поля

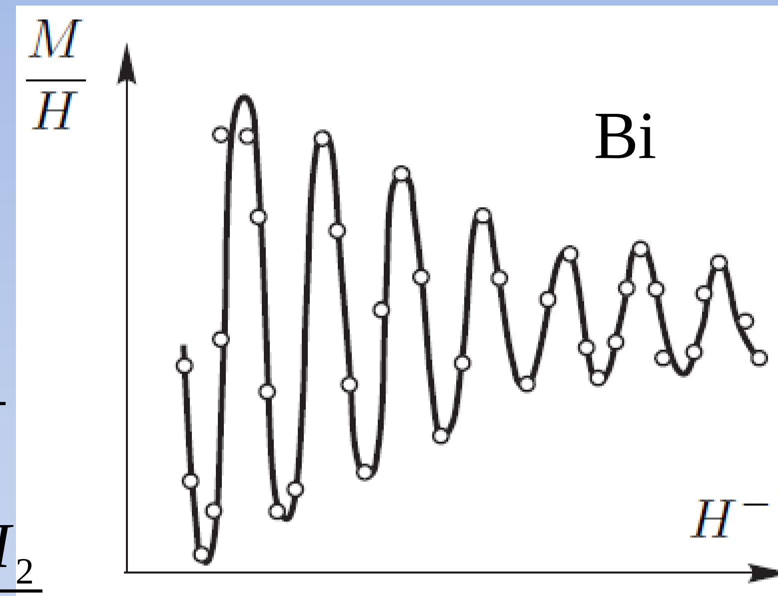
Эффект де Гааза – ван Альфена

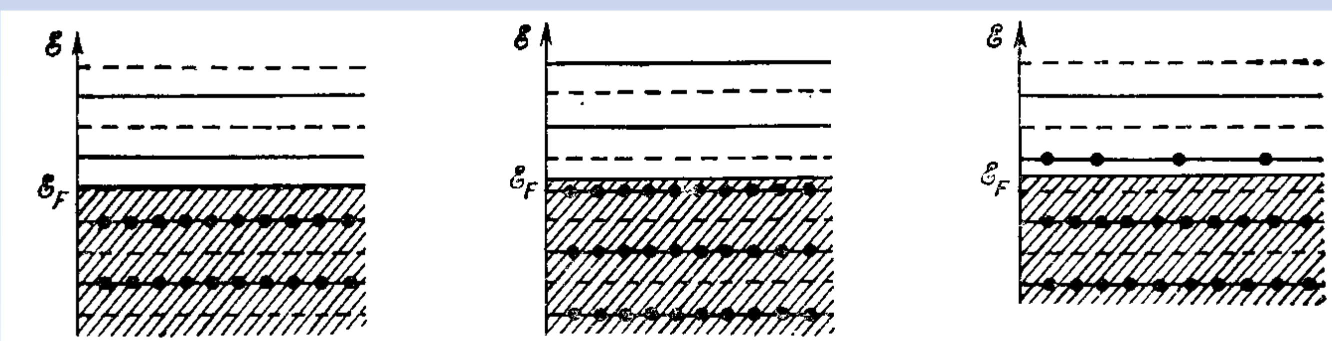


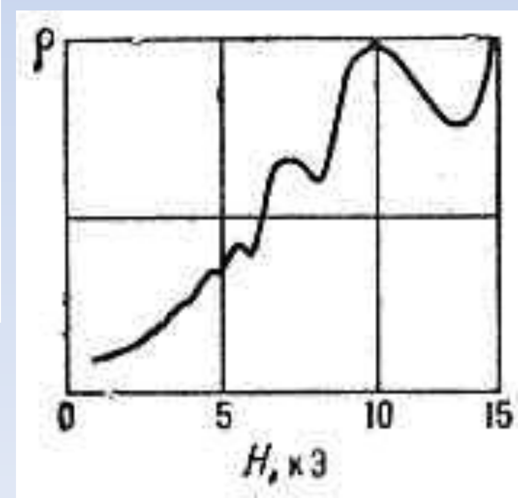
$$M \sim \sin\left(\frac{c\hbar S_m}{eH}\right)$$

$$S_m = \pi k_n^2 = 2\pi\left(n + \frac{1}{2}\right)\frac{eH_1}{\hbar c}$$

$$S_m = \pi k_{n-1}^2 = 2\pi\left(n - \frac{1}{2}\right)\frac{eH_2}{\hbar c}$$







Эффект Шубникова – де Гааза