

UCN and fundamental physics

V.F.Ezhov

Time Since Big Bang

Major Events Since Big Bang

present

Era of Galaxies

1 billion years

Era of Atoms

500,000 years

Era of Nuclei

3 minutes

Era of Nucleosynthesis

0.001 seconds

Particle Era

10^{-10} seconds

Electroweak Era

10^{-38} seconds

GUT Era

elementary particles

????

10^{-43} seconds

Planck Era

neutron
proton



electron
neutrino



antiproton
antineutron



antielectrons



quarks



stars,
galaxies
and clusters
(made of
atoms and
plasma)

atoms and
plasma
(stars
begin
to form)

plasma of
hydrogen and
helium nuclei
plus electrons

protons, neutrons,
electrons, neutrinos
(antimatter rare)

elementary particles
(antimatter
common)

elementary
particles

Humans
observe
the cosmos.

First galaxies
form.

Atoms form;
photons fly free
and become
microwave
background.

Fusion ceases;
normal matter is
75% hydrogen,
25% helium, by
mass.

Matter annihilates
antimatter.

Electromagnetic and weak
forces become distinct.

Strong force becomes
distinct, perhaps
causing inflation of
universe.

Requirements for the Sakharov Process

1. *The process must violate Baryon Number Conservation*
2. *There must be a period of Non-Thermal Equilibrium*
3. *There must be a process that violates
Time Reversal Non-Invariance --- "T-violation"*



A. Sakharov

Generating a Matter-Antimatter Asymmetry

A. D. Sakharov, JETP Lett. 5, 24 (1967).

1. Very early in the Big Bang ($t < 10^{-6}$ s), matter and antimatter (i.e. p & \bar{p}) were in thermal equilibrium ($T \gg 1$ GeV). There was exact balance between matter and antimatter.
2. At some point, there was a symmetry breaking process that led to a small imbalance between the number of Baryons and Anti-Baryons...i.e a few more Baryons.
3. When the Universe cooled to below $T \sim 1$ GeV, All the anti-baryons annihilated leaving a few baryons and lots of high-energy annihilation photons.
4. The photons are still around! They have been highly red shifted by subsequent expansion and are now microwaves as the Cosmic Microwave Background.

In this scenario, the total “apparent” matter-antimatter asymmetry is really very tiny... given by ratio of Baryons to CMB photons:

$$\frac{n_{\text{Baryon}}}{n_{\gamma}} \approx 10^{-10}$$

Theories of CP-violation to experiment

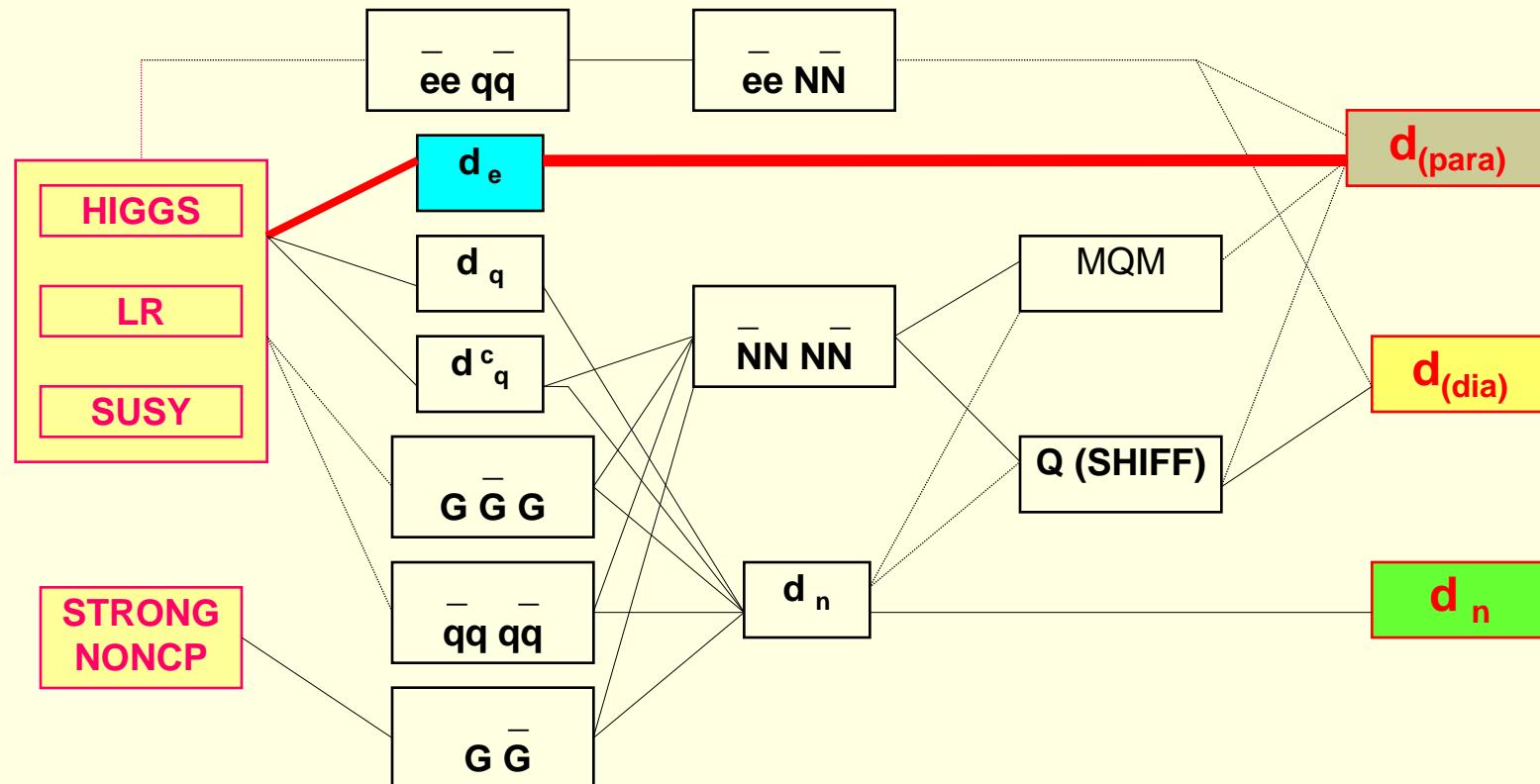
Fundamental Theory

CP violating interactions

Nucleon-nucleon & nucleon-electron interactions

Nuclear structure

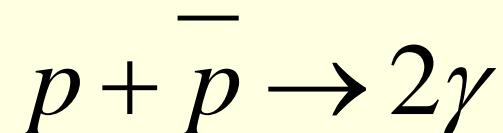
Atomic & molecular structure

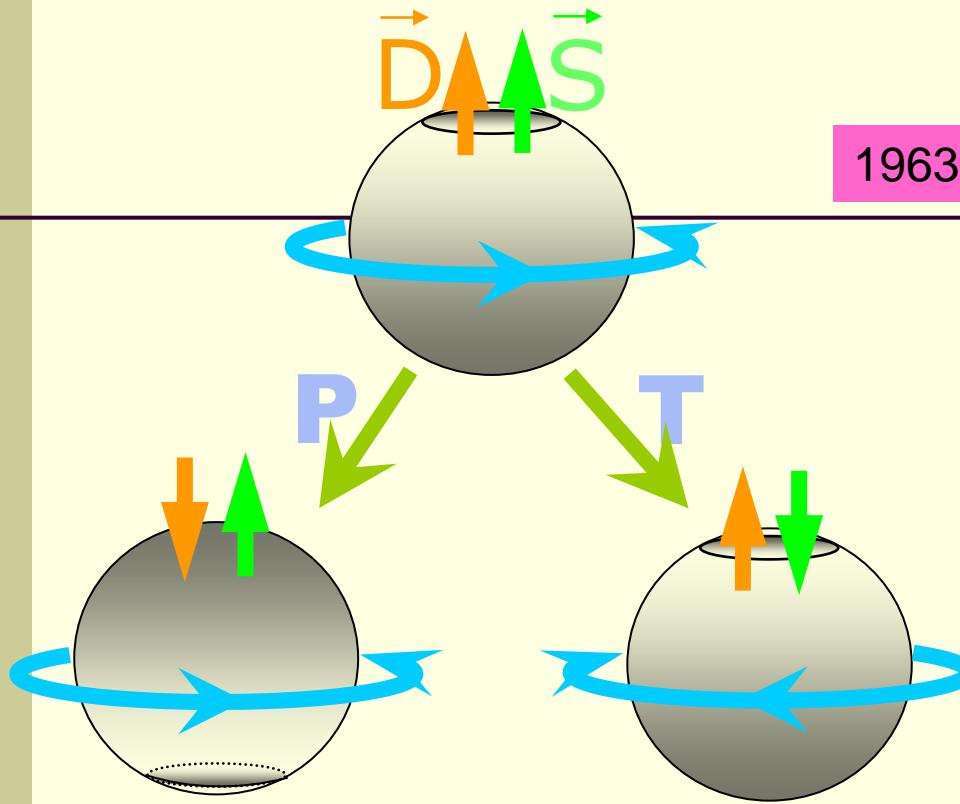


The current limit $d_e < 1.6 \times 10^{-27} e \cdot cm$ has already discounted many variants of Supersymmetry

Барионная асимметрия

- При температуре $5 \cdot 10^{12}$ К закончилась *стадия рекомбинации*: почти все протоны и нейтроны аннигилировали, превратившись в фотоны; остались только те, для которых не хватило античастиц. Как показали наблюдения, на один барион приходится почти миллиард фотонов – продуктов аннигиляции. Значит, первоначальный избыток частиц по сравнению с античастицами составляет одну миллиардовую от их числа. Именно из этого «избыточного» вещества и состоит в основном вещество наблюдаемой Вселенной.





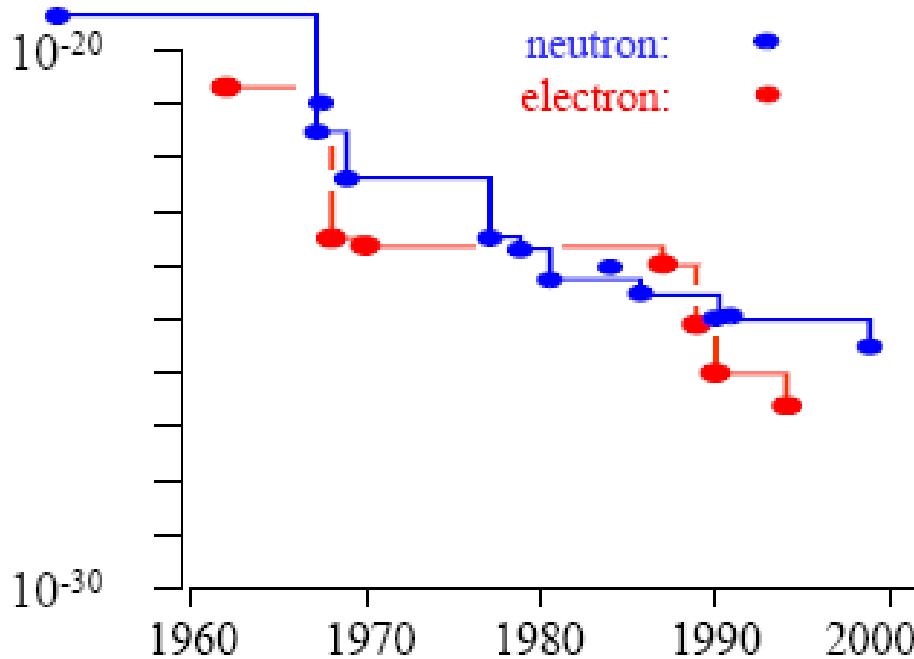
1963 – CP-violation in K^0 -decay

- System must be neutral one.
- NMR resonance shift correlated to electric field is measured.

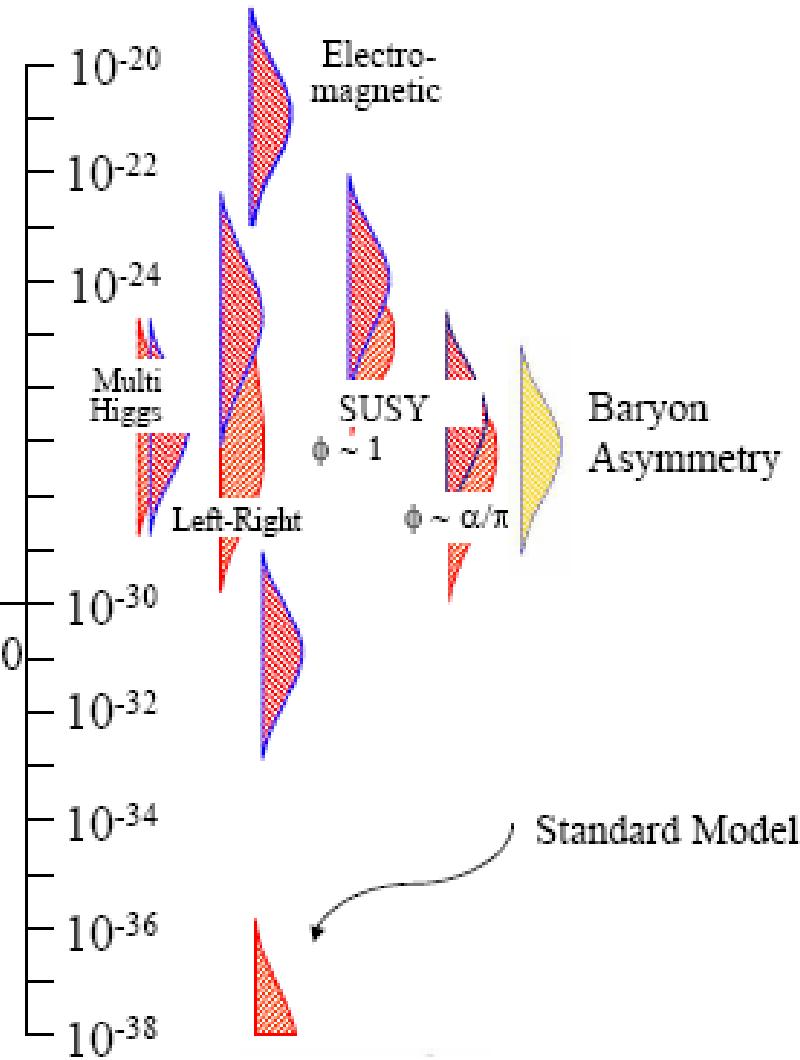
$$H = -(\mu \vec{B} + d \vec{E}) \cdot \frac{\vec{F}}{|F|}$$

EDM limits: the first 50 years

Experimental Limit on d (e cm)

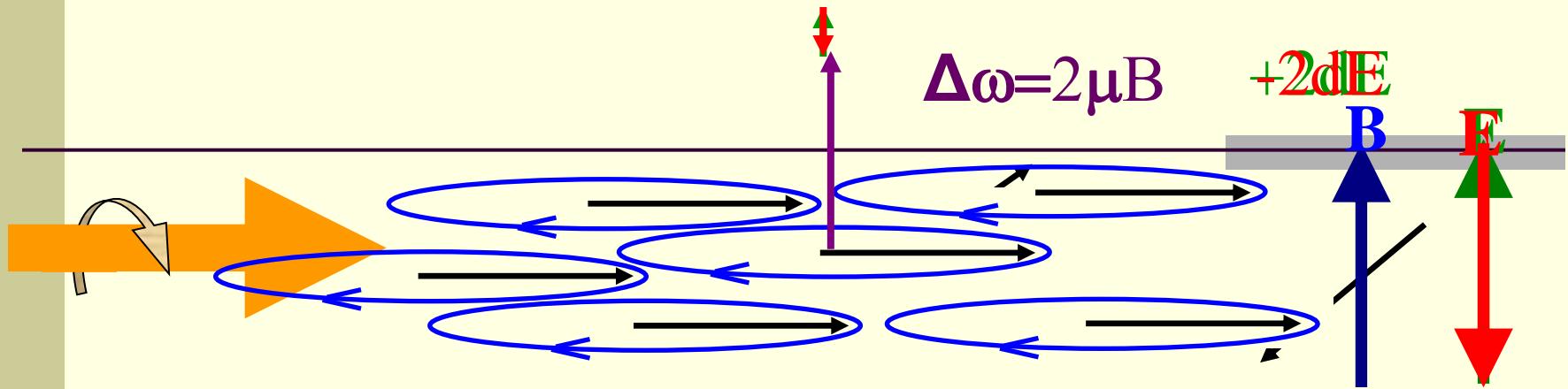


neutron:
electron:

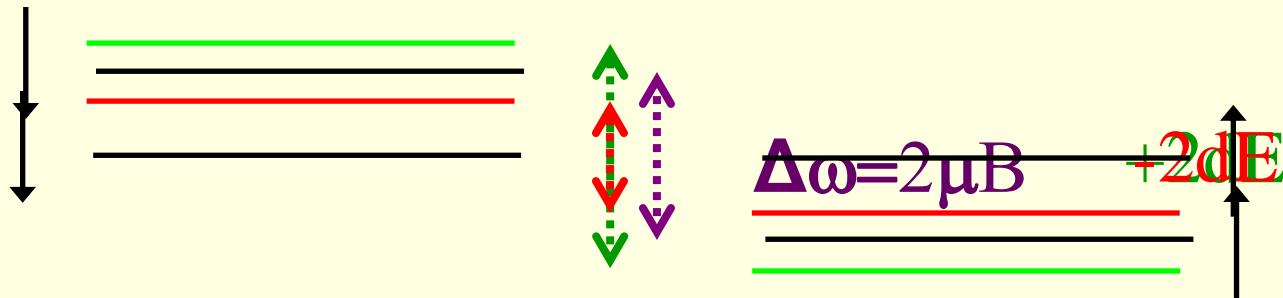


Updated from Barr: Int. J. Mod Phys. A8 208 (1993)

General method to detect an EDM



Energy level picture:



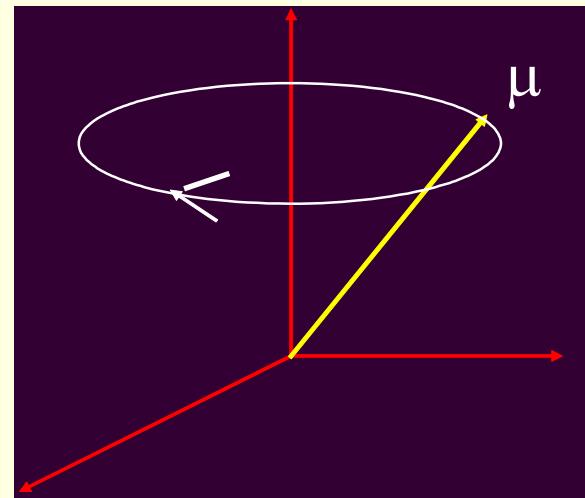
Neutron beam
 $\Delta\omega \cdot \Delta t \approx h$ $\rightarrow \Delta\omega \rightarrow 0 \text{ & } \Delta t \rightarrow \infty$
UCN storage

Solution

Larmor Equation - the basic NMR equation

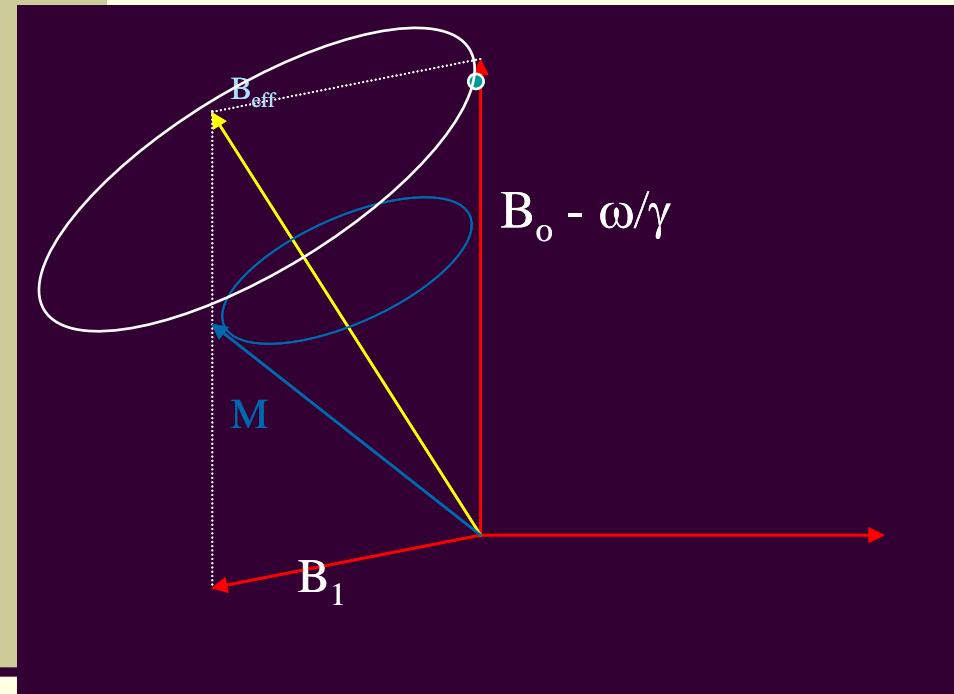
$$\frac{d\mu}{dt} = \gamma \mu \times B$$

$$\omega = -\gamma B$$



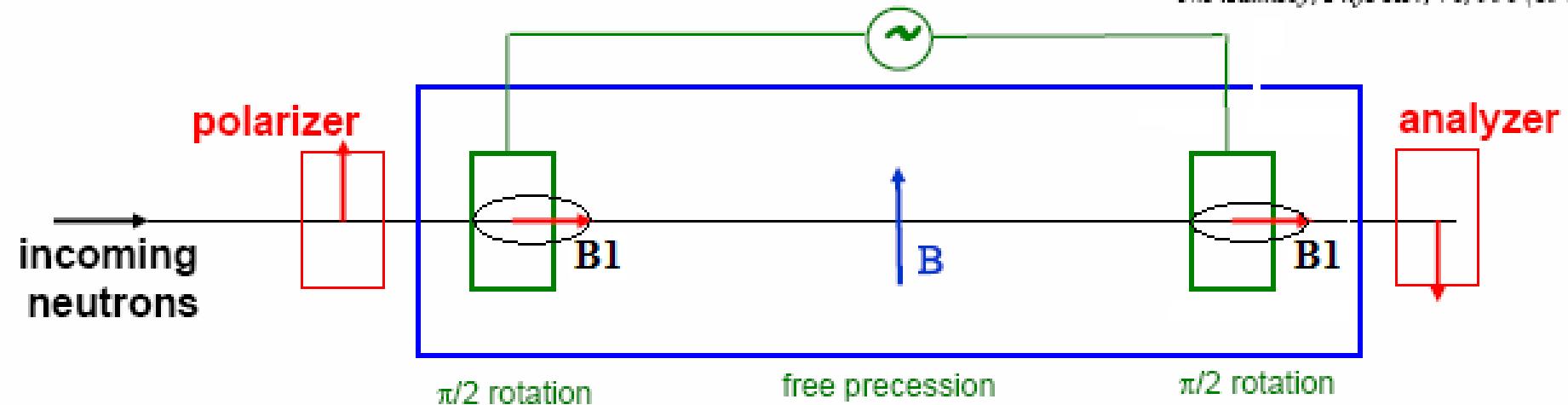
$$\frac{dM}{dt} = \gamma M \times B_{eff}$$

$$B_{eff} = B_1 \vec{i} + (\omega - \omega_0) / \gamma \vec{k}$$

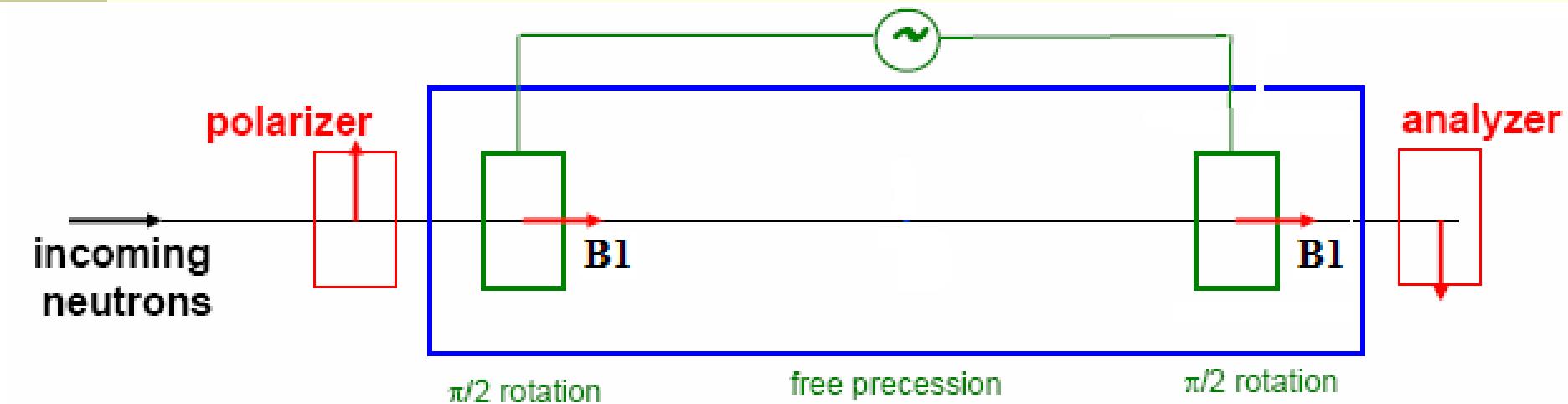


Method of Separated Oscillatory Fields

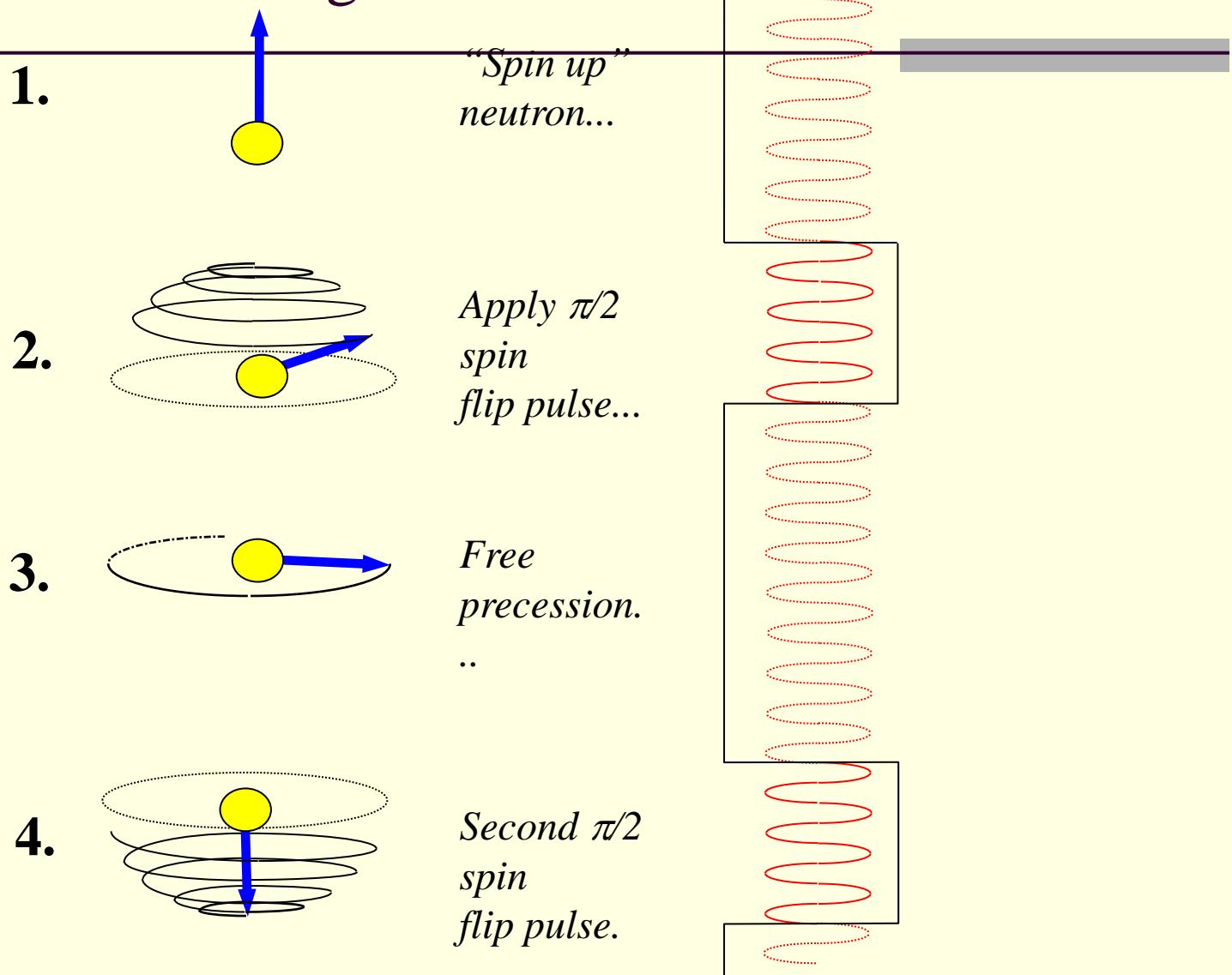
N.F.Ramsey, Phys Rev, 76, 996 (1949)

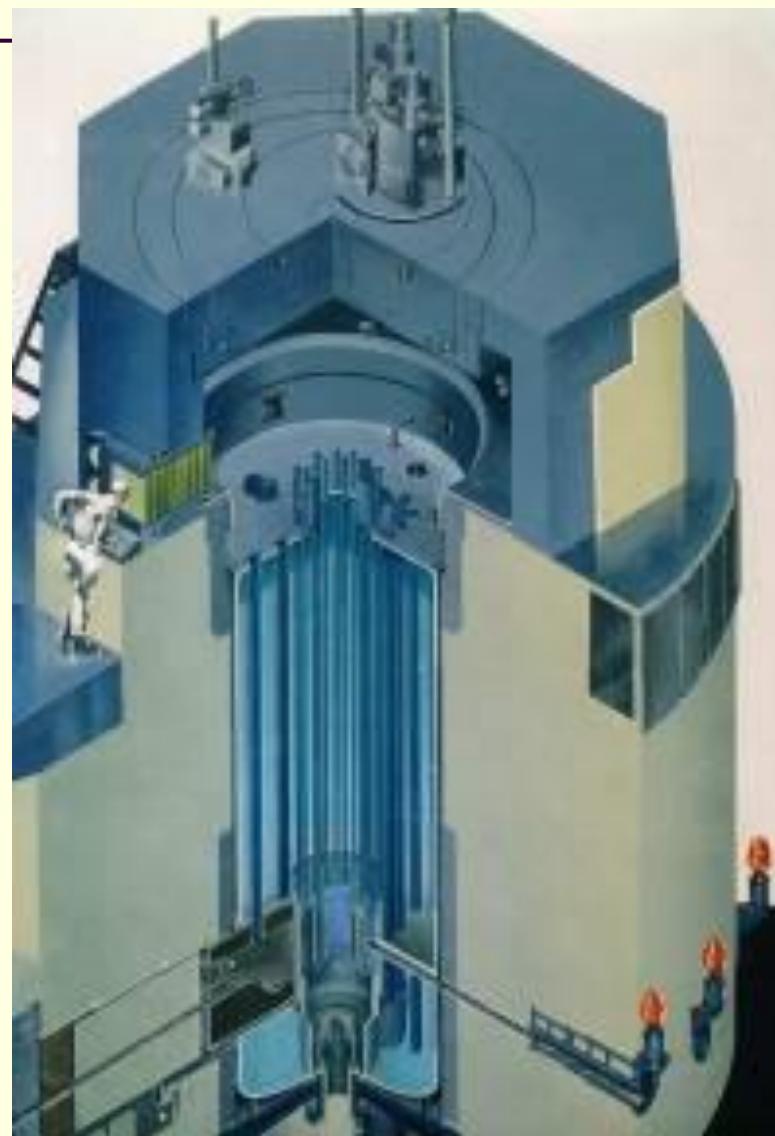


Rotational reference

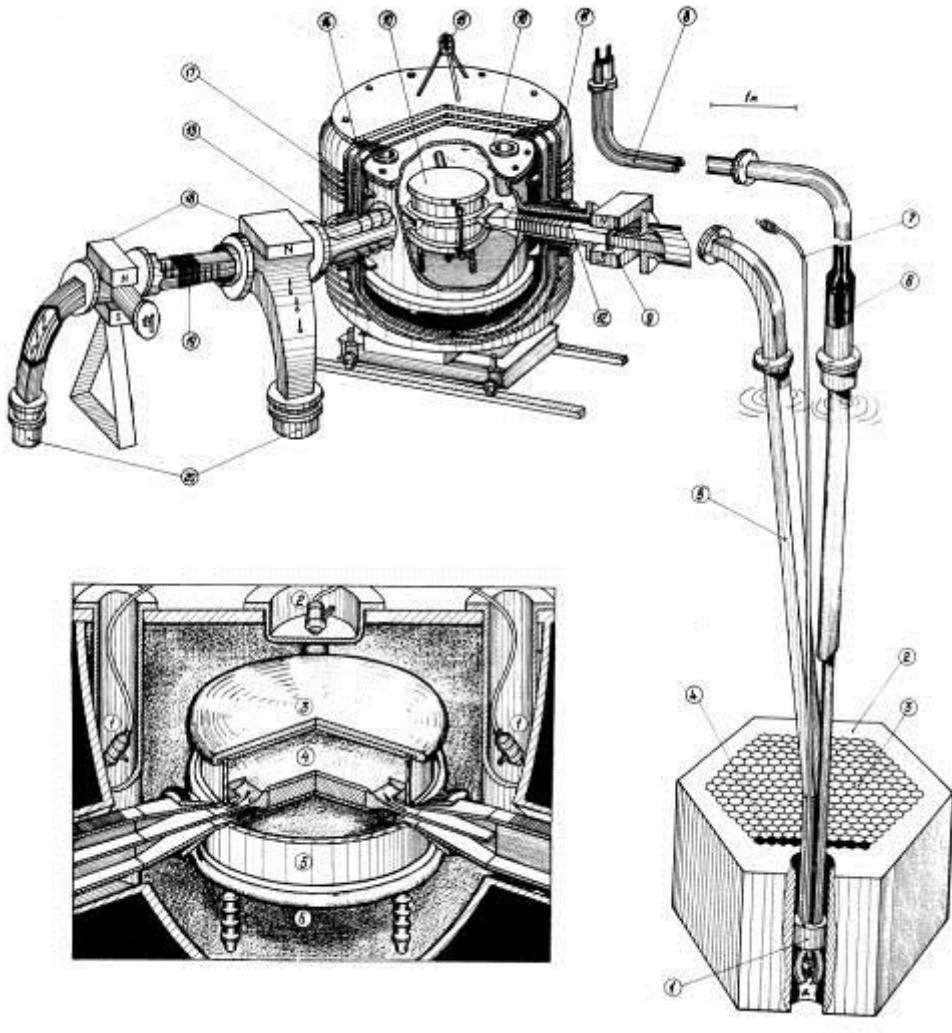


Ramsey method of Separated Oscillating Fields



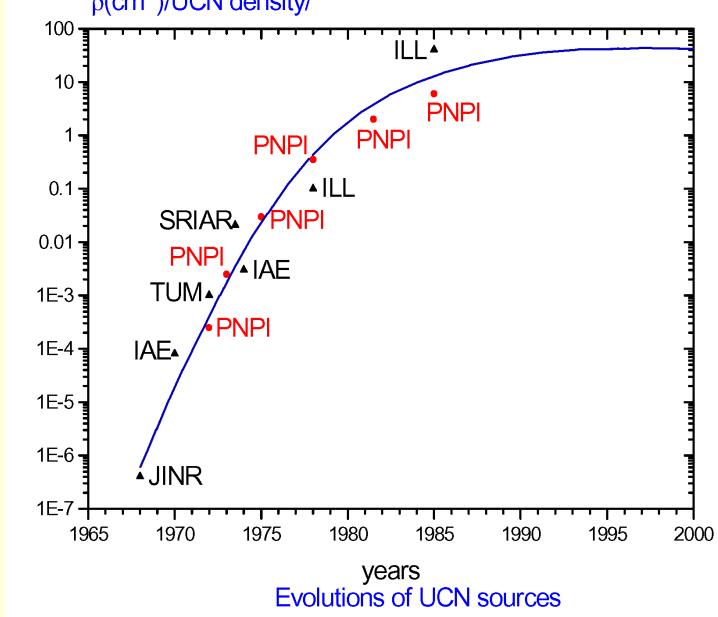
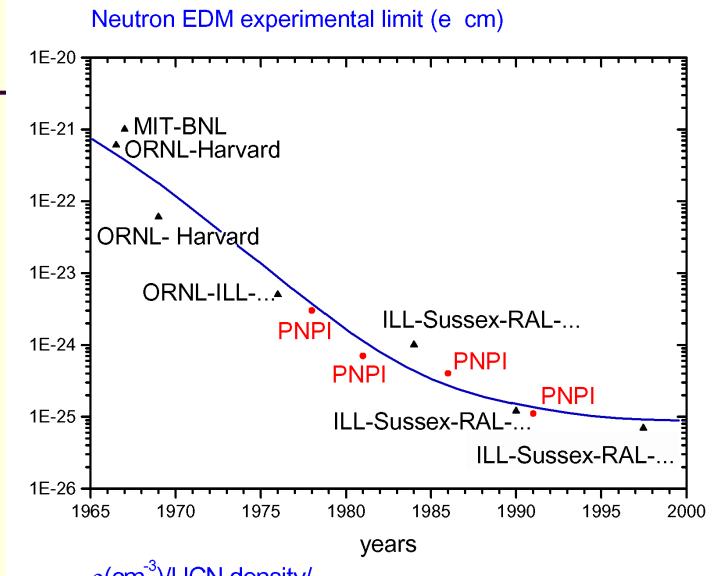


The neutron EDM experiment at PNPI (1970-1995)

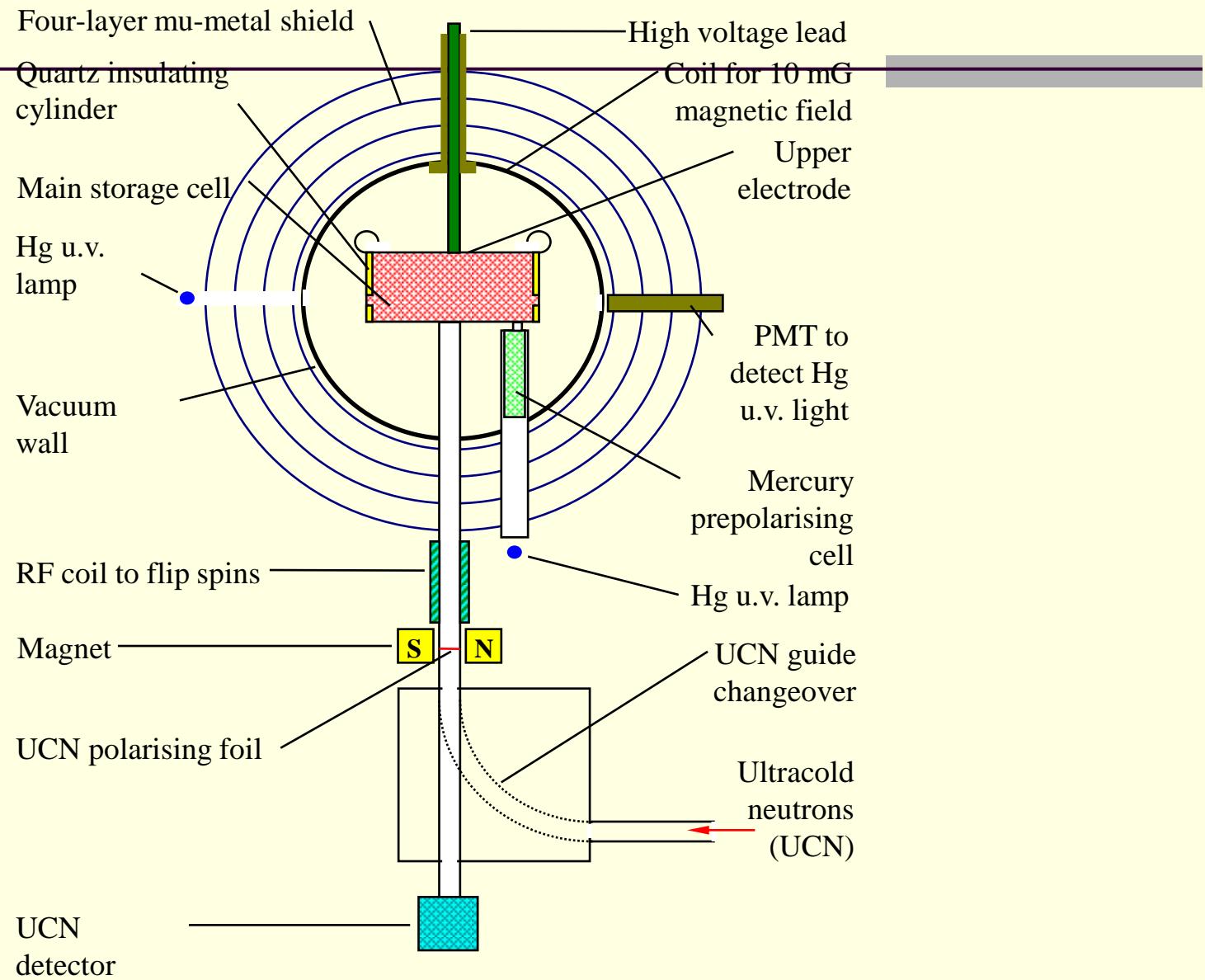


$$d_n = (+2.6 \pm 4.0 \pm 1.6) \cdot 10^{-26} \text{ e}\cdot\text{cm}$$

$$|d_n| < 9.7 \cdot 10^{-26} \text{ e}\cdot\text{cm} \text{ (90% C.L.)}$$



nEDM apparatus

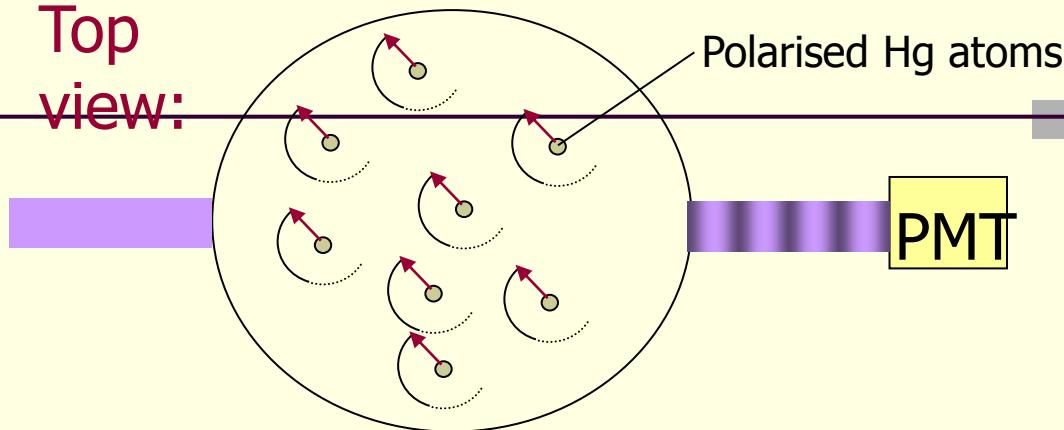


Experimental setup

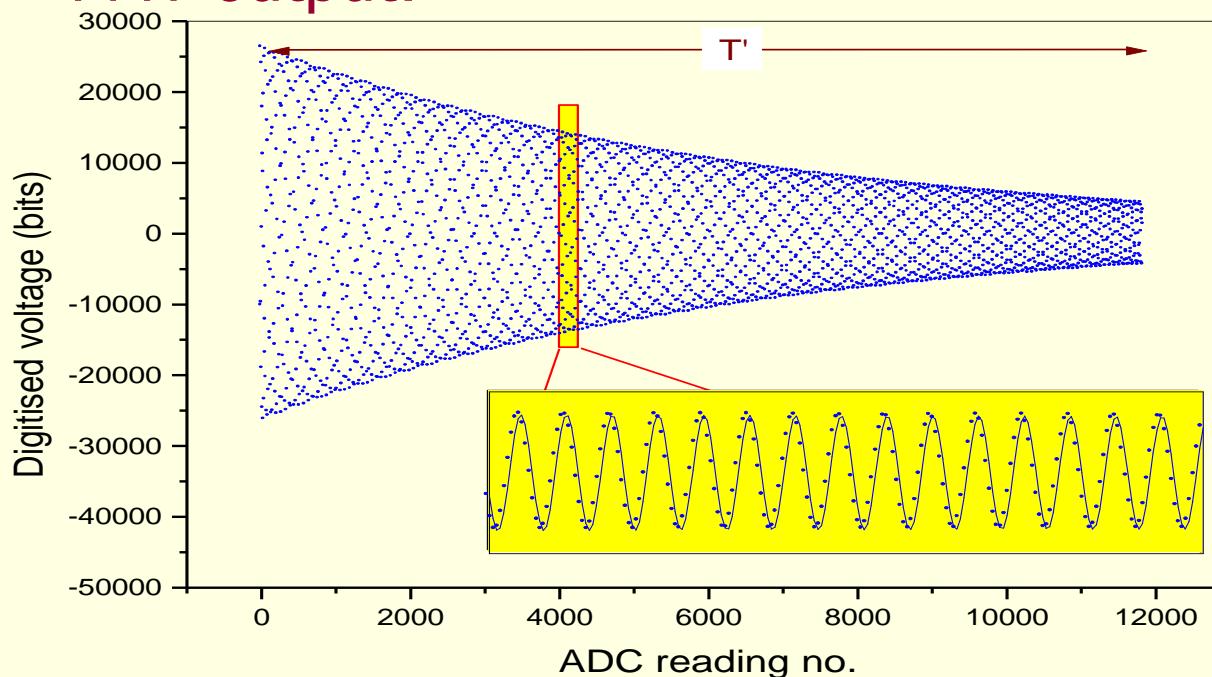


Hg co-magnetometer

Top view:



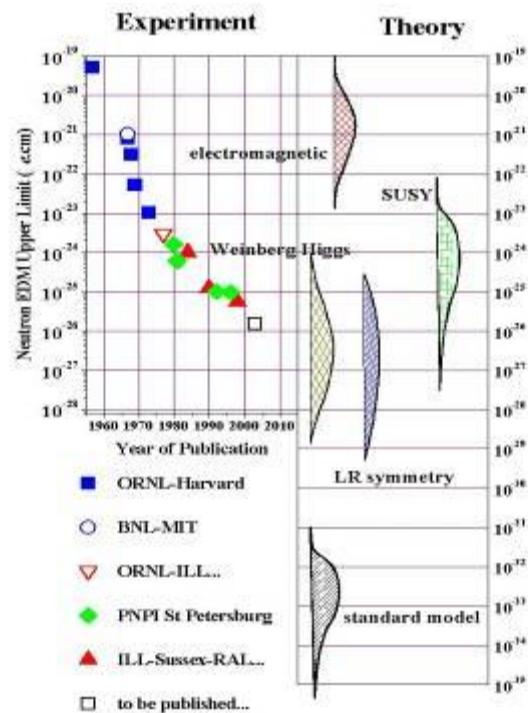
PMT output:



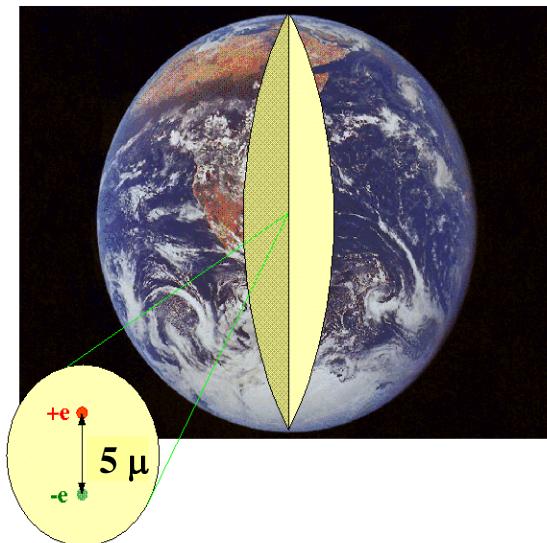
$$d_n = (-0.31 \pm 1.54 \pm 1.00) \times 10^{-26} \text{ e.cm}$$

New limit:
 $|d_n| < 3.1 \times 10^{-26} \text{ e.cm (90\% CL)}$

Comparison with theory



If a neutron were the size of the Earth...



... current EDM limit would correspond to
charge separation of $\approx 5 \mu$

Multi-chamber EDM Spectrometer. Status 2005*

E. Aleksandrov⁴, M. Balabas⁴, Yu. Borisov¹, S. Dmitriev³, N. Dovator³, O. Dymshits⁵, V. Ezhov¹,
A. Fomin¹, P. Geltenbort², P. Iaydjiev⁶, A. Ivanov⁴, S. Ivanov¹, V. Kartoshkin³, M. Karuzin⁴,
A. Kharitonov¹, A. Khusainov¹, E. Koshurnikov¹, I. Kotina¹, I. Krasnoshekova¹, V. Kulyasov⁴,
M. Lasakov¹, O. Lodkina¹, V. Marchenkov¹, A. Murashkin¹, A. Pazgalev⁴, A. Pikalev⁷, A. Pustovoit¹,
T. Savelieva¹, M. Sazhin¹, S. Sbitnev¹, A. Serebrov¹, A. Shashkin⁵, G. Shmelev¹, I. Shoka¹,
E. Siber¹, V. Solovei¹, M. Svinin⁷, R. Taldaev¹, I. Tokmakov⁷, V. Varlamov¹, A. Vasiliev¹, A. Zhilin⁵

¹PNPI, St. Petersburg Nuclear Physics Institute, Gatchina, Russia

²ILL, Institut Laue-Langevin, Grenoble, France

³Ioffe Physical Technical Institute, Russ. Acad. Sc., St. Petersburg, Russia

⁴Vavilov State Optical Institute, St. Petersburg, Russia

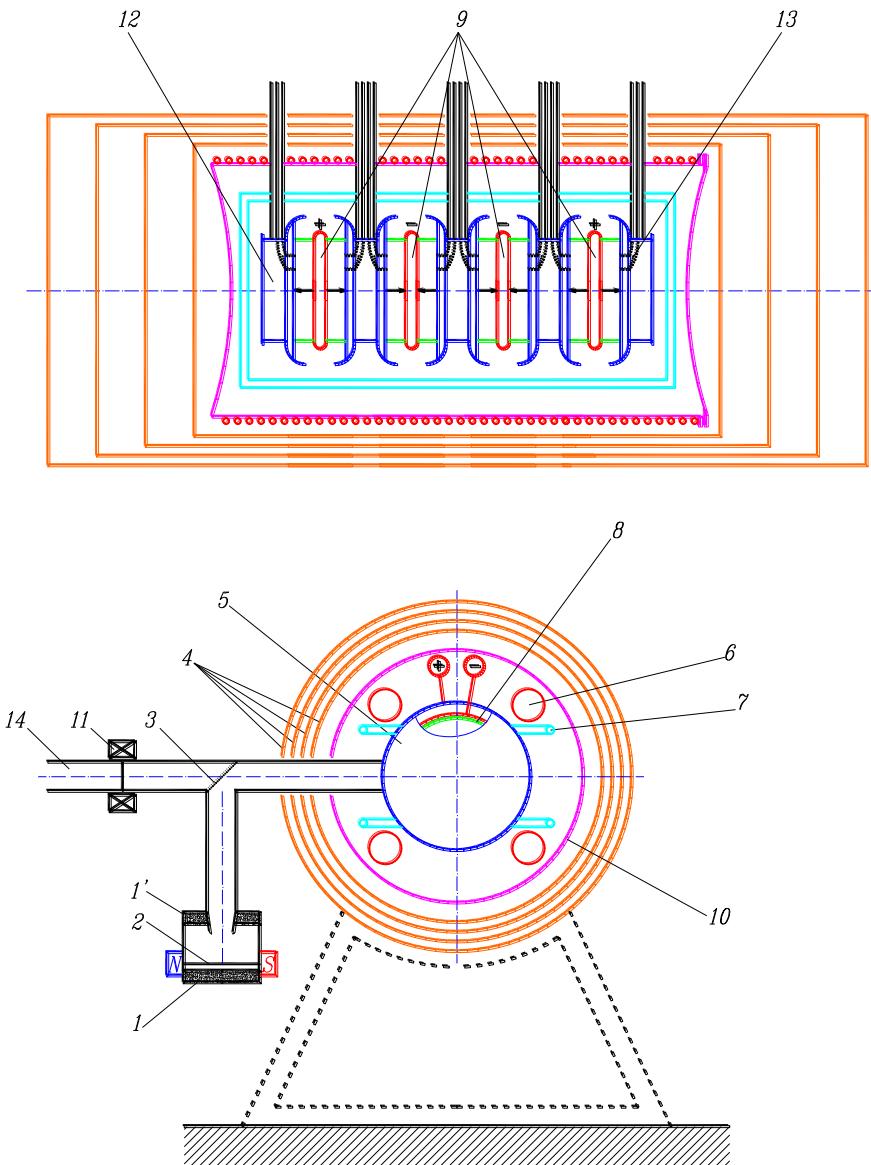
⁵Scientific Research Institute of Optical Materials Technology, St. Petersburg, Russia

⁶Sofia Nuclear Research Institute, Sofia, Bulgaria

⁷Efremov Research Institute, St. Petersburg, Russia

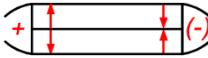
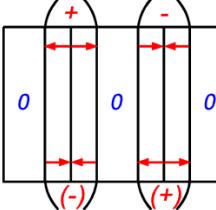
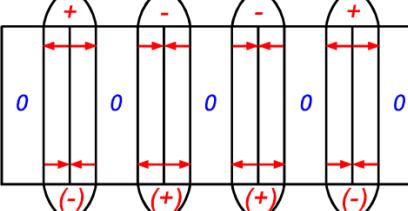
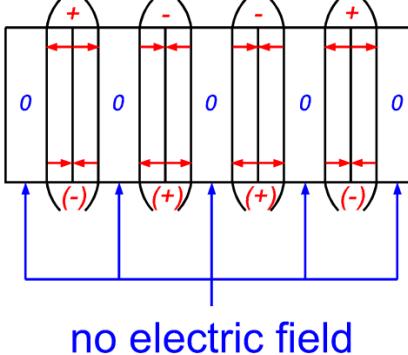
* In connection with that some part of participants left collaboration we are publishing new present list of participants. We are thankful to our colleagues for useful discussions and wish them further success

The scheme of the multichamber nEDM spectrometer



1. 1' - UCN detectors
2. - polarization analyzer foil
3. - UCN switch
4. - four-layer magnetic shield
5. - electrode with zero potential
6. - channel for Cs magnetometers
7. - oscillating field coils
8. - BeO-coated insulator
9. - HV electrodes
10. - vacuum chamber with magnetic field coil
11. - superconducting polarizer with a membrane to separate the vacuum of the UCN source from the vacuum of the EDM spectrometer
12. - UCN storage chamber
(1 out of 13)
13. - UCN shutter
14. - UCN guide

Multichamber nEDM spectrometer and its advantages

1.		Double chamber scheme compensates $\sigma_0(t)$ only
2.		Two double chamber scheme with opposite HV polarity $(- +)$ compensates $\sigma_0(t) + \sum_z \sigma_{1z}(t) \cdot z$
3.		Four double chamber scheme with HV polarity $(- + + -)$ compensates $\sigma_0(t) + \sum_z \sigma_{1z}(t) \cdot z + \sum_z \sigma_{2z}(t) \cdot z^2$
4.		Four double chamber scheme with HV polarity $(- + + -)$ and neutron comagnetometers compensates $\sigma_0(t) + \sum_z \sigma_{1z}(t) \cdot z + \sum_z \sigma_{2z}(t) \cdot z^2 + \sum_z \sigma_{3z}(t) \cdot z^3 + \dots ?$

Data analysis with a multichamber EDM spectrometer

EDM and magnetic fluctuations

1.	$\bar{I} = \frac{1}{8} \{ [(F_2 - F_3) - (F_5 - F_6)] - [(F_8 - F_9) - (F_{11} - F_{12})] \}$	$(\Delta\delta F)_L - (\Delta\delta F)_R = D$	neutron EDM indication (from HV-traps only!)
2.	$\bar{I}^+ = \frac{1}{8} \{ [(F_2 - F_3) - (F_5 - F_6)] + [(F_8 - F_9) - (F_{11} - F_{12})] \}$	$(\Delta\delta F)_L + (\Delta\delta F)_R$	fluctuations with terms of 2 nd , 4 th , etc. orders
3.	$\bar{II}^+ = \frac{1}{8} \{ [(F_2 - F_3) + (F_5 - F_6)] + [(F_8 - F_9) + (F_{11} - F_{12})] \}$	δF	fluctuations with terms of 1 st , 3 rd , etc. orders
4.	$\bar{III}^+ = \frac{1}{8} \{ [(F_2 + F_3) + (F_5 + F_6)] + [(F_8 + F_9) + (F_{11} + F_{12})] \}$	F	fluctuations of uniform magnetic field, 2 nd , etc. orders

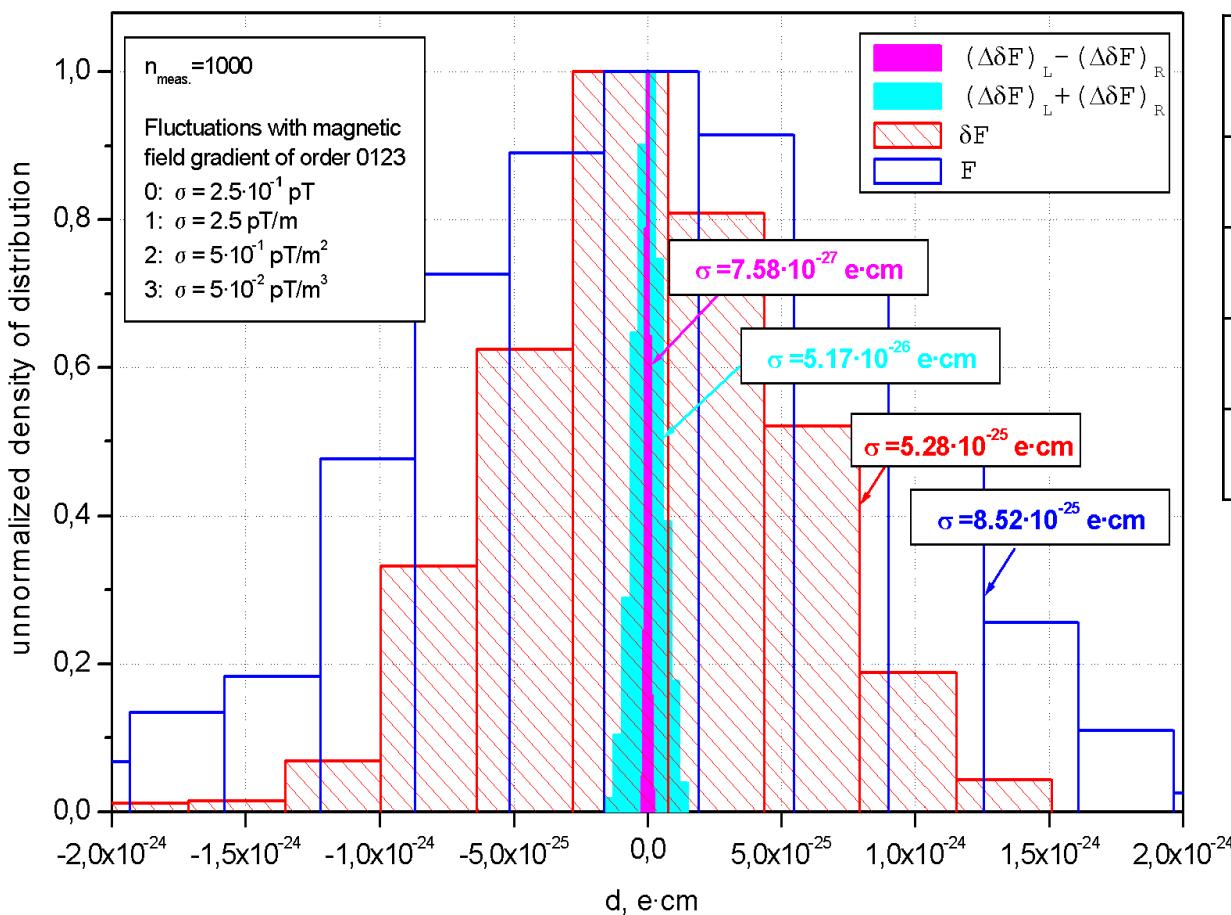
Magnetic field distribution

5.	$\bar{VIII}^- = \frac{1}{8} \{ [(B_2 - B_3) - (B_5 - B_6)] - [(B_8 - B_9) - (B_{11} - B_{12})] \}$	$(\Delta\delta B)_L - (\Delta\delta B)_R$	magnetic field terms of 3 rd , 5 th , etc. orders
6.	$\bar{VIII}^+ = \frac{1}{8} \{ [(B_2 - B_3) - (B_5 - B_6)] + [(B_8 - B_9) - (B_{11} - B_{12})] \}$	$(\Delta\delta B)_L + (\Delta\delta B)_R$	magnetic field terms of 2 nd , 4 th , etc. orders
7.	$\bar{VII}^+ = \frac{1}{8} \{ [(B_2 - B_3) + (B_5 - B_6)] + [(B_8 - B_9) + (B_{11} - B_{12})] \}$	δB	magnetic field terms of 1 st , 3 rd , etc. orders
8.	$\bar{V}^+ = \frac{1}{8} \{ [(B_2 + B_3) + (B_5 + B_6)] + [(B_8 + B_9) + (B_{11} + B_{12})] \}$	B	average value of the deviation from the resonance

EDM(n)=D-D₀

Compensation of magnetic field fluctuations by means of multichamber nEDM spectrometer and requirement for stability of magnetic field

The effect of compensation

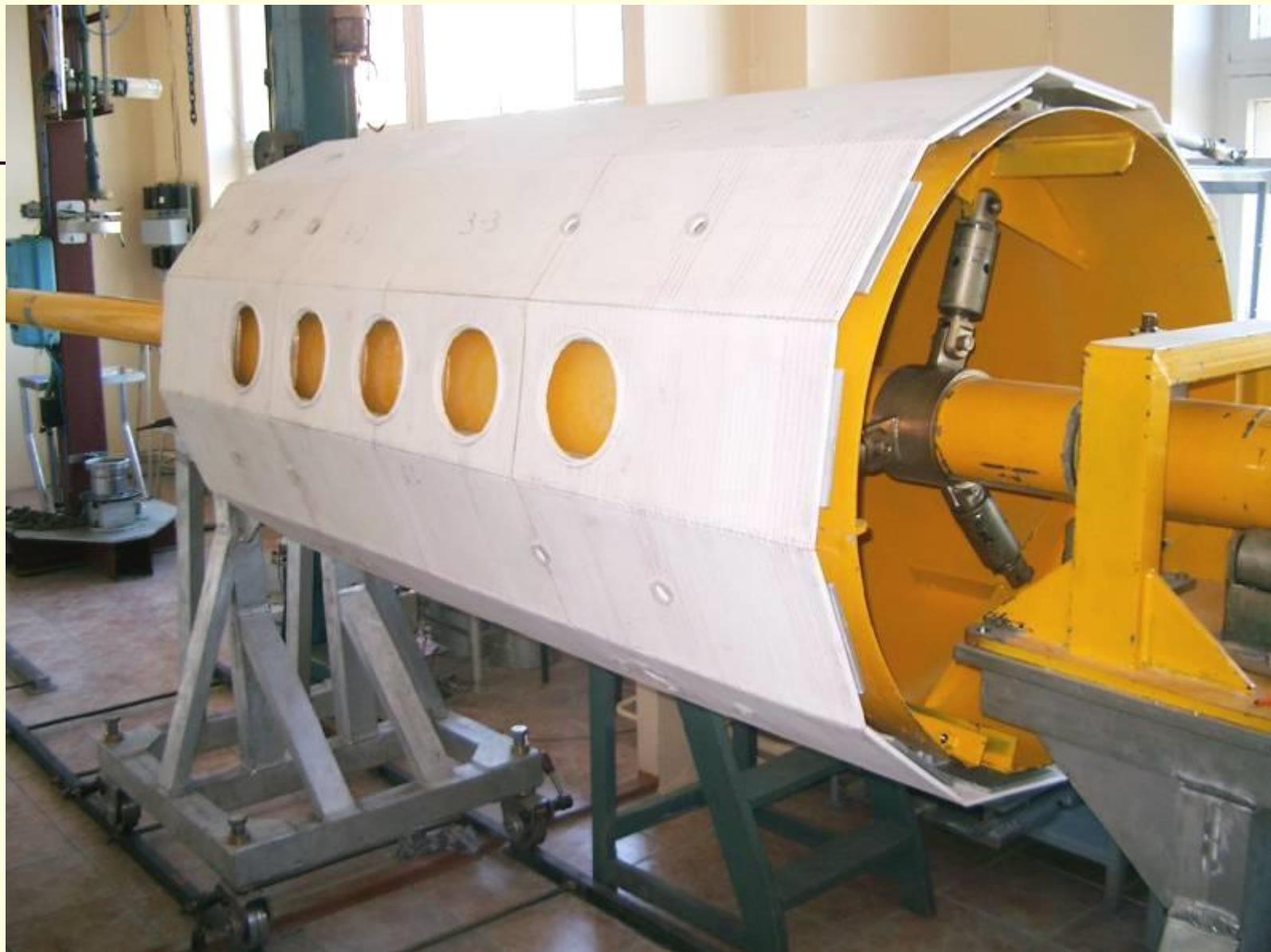


The requirement

fluctuation type	Version II final EDM	Version I mini EDM
σ_{0z}	1 pT	3.5 pT
σ_{1z}	10 pT/m	35 pT/m
σ_{2z}	1 pT/m ²	3.5 pT/m ²
σ_{3z}	0.1 pT/m ³	0.35 pT/m ³



Housing of neutron guide system and vacuum chamber

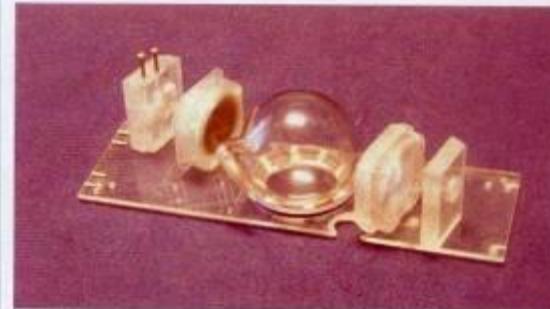
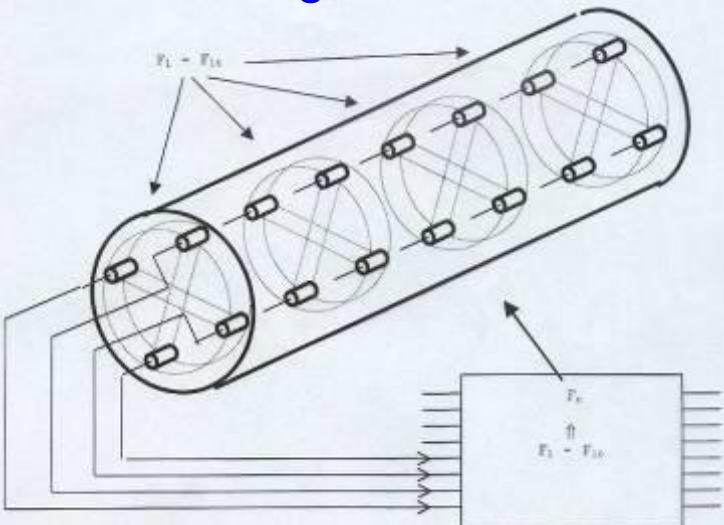


Quartz ceramic frame for magnetic field solenoid

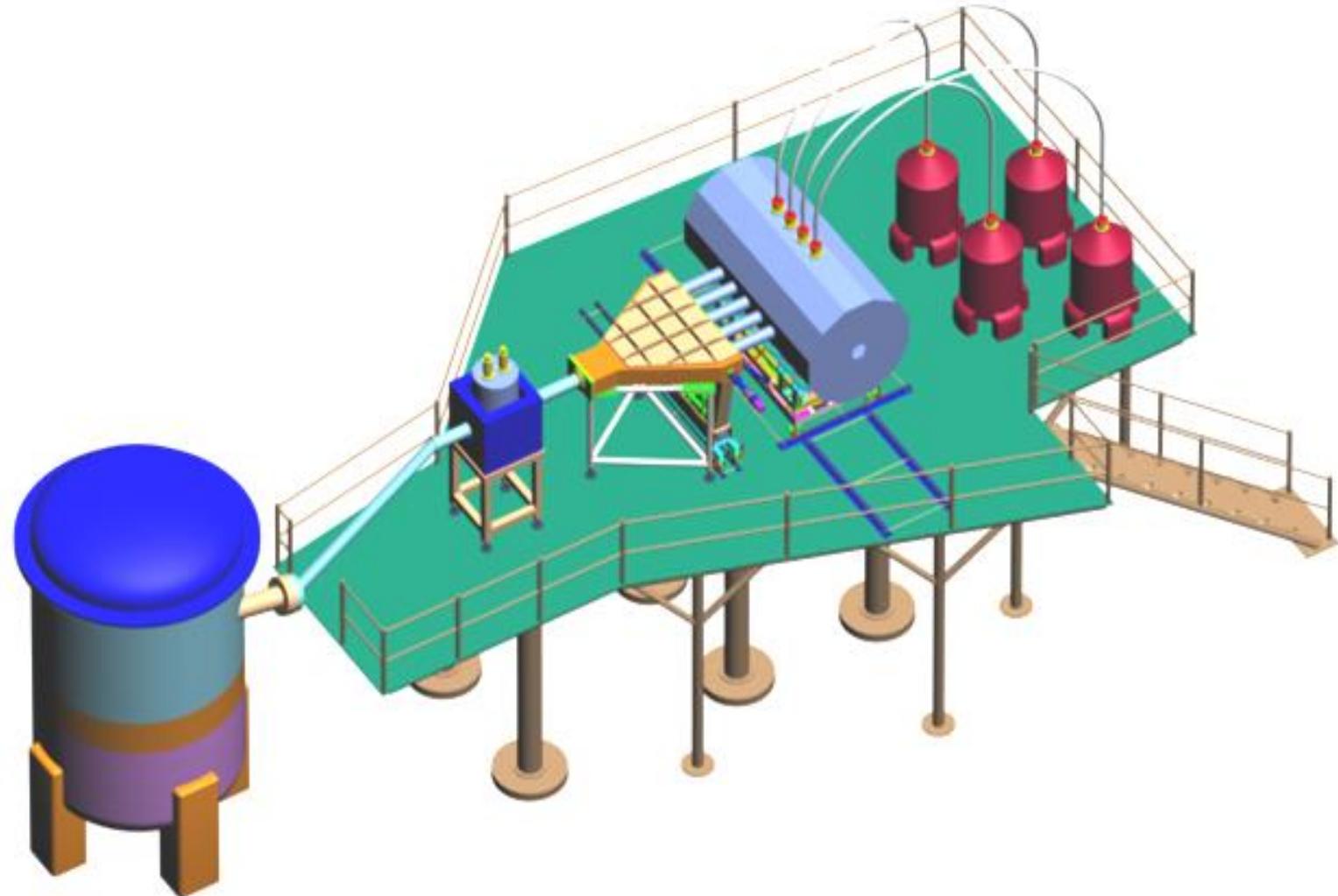
Cs-magnetometers

Preparation of 16 Cs-magnetometers at IPTI and VSOI

The scheme of
16 Cs-magnetometers



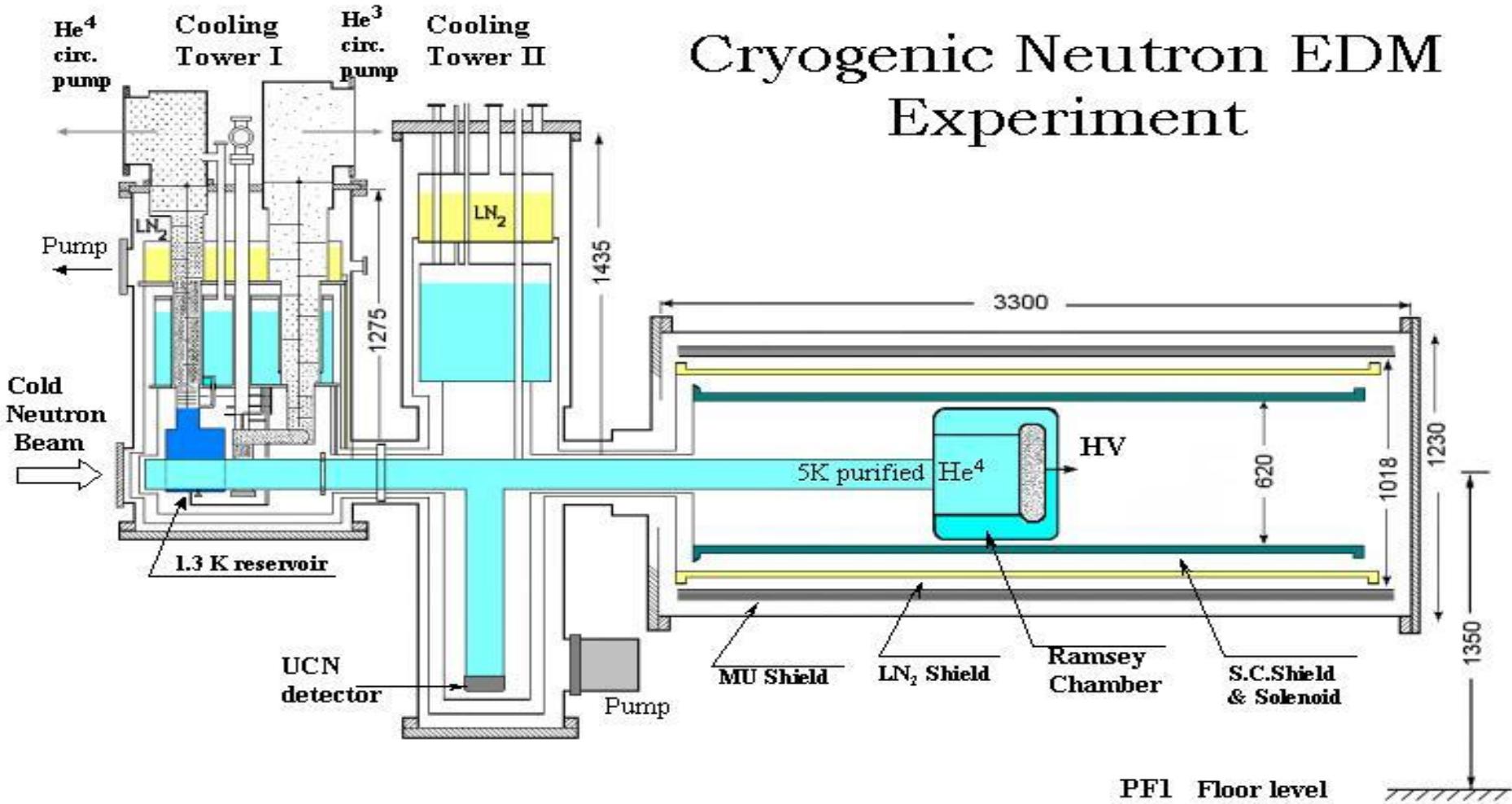
Project accuracy of measurement $< 10^{-26} \text{ e}\cdot\text{cm} / 100 \text{ days}$



Scheme of allocation of multi-chamber EDM spectrometer at PF2 UCN facility

Next generation

Cryogenic Neutron EDM Experiment



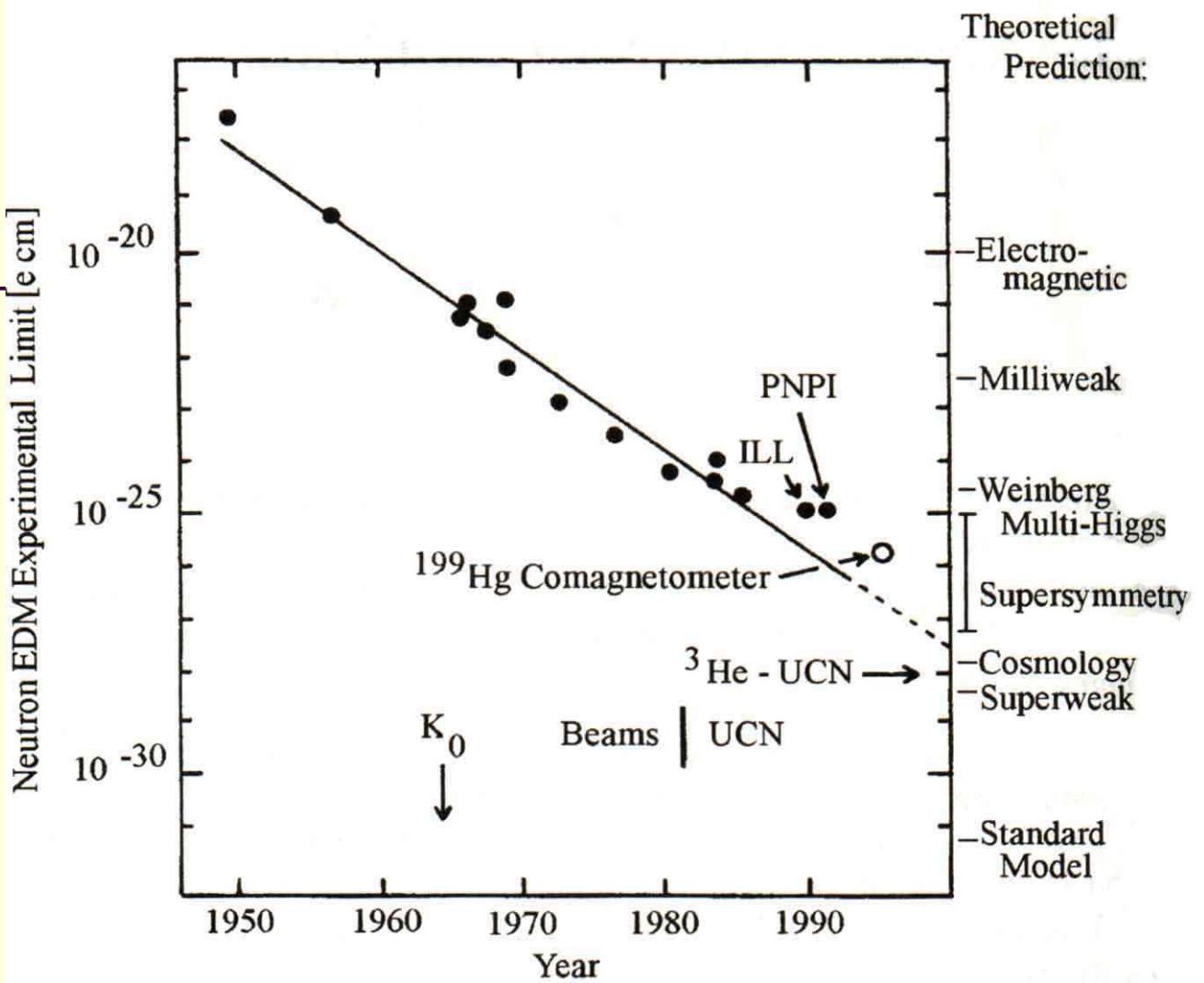


Fig. 4.1. Historical development of the neutron EDM experimental limit along with some theoretical predictions

$$\Delta\omega \approx 10^{-8} s^{-1}$$

$$\delta d = \frac{\Delta\omega}{E}$$

Physics model	$ d_e $
Standard Model	$\sim 10^{-41} \text{ e}\cdot\text{cm}$
Left-right symmetric	$10^{-26}\text{-}10^{-28} \text{ e}\cdot\text{cm}$
Lepton flavor-changing	$10^{-26}\text{-}10^{-29} \text{ e}\cdot\text{cm}$
Multi-Higgs	$10^{-27}\text{-}10^{-28} \text{ e}\cdot\text{cm}$
Technicolor	$\sim 10^{-29} \text{ e}\cdot\text{cm}$
Supersymmetry	$< 10^{-25} \text{ e}\cdot\text{cm}$

$$|d_e| < 1.6 \times 10^{-27} \text{ e}\cdot\text{cm}$$

B. Regan, E. Commins, C. Schmidt,
D. DeMille, PRL **88**, 071805 (2002)

EDM enhancement factor

- **Shiff theorem:** In atoms and molecules an electric field on e^- and on nuclear Z_n^+ is equal to zero.
- Non zero electric field appears due to magnetic (relativistic) effects.

$$\vec{F}_{tot} = \vec{F}_{el} + \vec{F}_{mag} = -e\vec{E}_{int} - e\frac{\vec{v}}{c} \times \vec{B} = 0$$

Relativistic effects increase with Z and internal electric field can be more than external.

$$\vec{B} = \frac{\vec{v}}{c} \times \vec{E}_{Z_n} \approx \frac{\vec{v}}{c} \times \frac{Ze}{r^2} \hat{r}$$

$$\vec{E}_{int} \approx \alpha^2 \frac{Z^3 e}{r^2} \hat{r}$$

$$v \approx Zac$$

$$\Delta E = d_a E_{ext} = R d_{e(Z_n)} E_{int}$$

EDM enhancement factor

- Energy shift for external electric field polarized paramagnetic atom or molecule:

$$\Delta E_i \approx 2\langle \Psi_i | d_e Z^3 \alpha^2 \frac{e}{r^2} \vec{\sigma} \cdot \vec{r} | \Psi_j \rangle \left[\frac{\langle \Psi_j | e \vec{r} \cdot \vec{E}_{ext} | \Psi_i \rangle}{E_i - E_j} \right] \approx d_e \bullet Z^3 \alpha^2 \frac{e}{a_0^2} \bullet P$$

- Polarization $\approx d_e \bullet E_{int} \bullet P$ (d_e is spherical)

$$P \sim E_{St}/Ry \sim 10^{-3}$$

- Polarization of molecule

$$P \sim E_{St}/E_{rot} \text{ or } E_{St}/E_A \sim 1$$

- For diamagnetic molecule (electric field at nuclear)

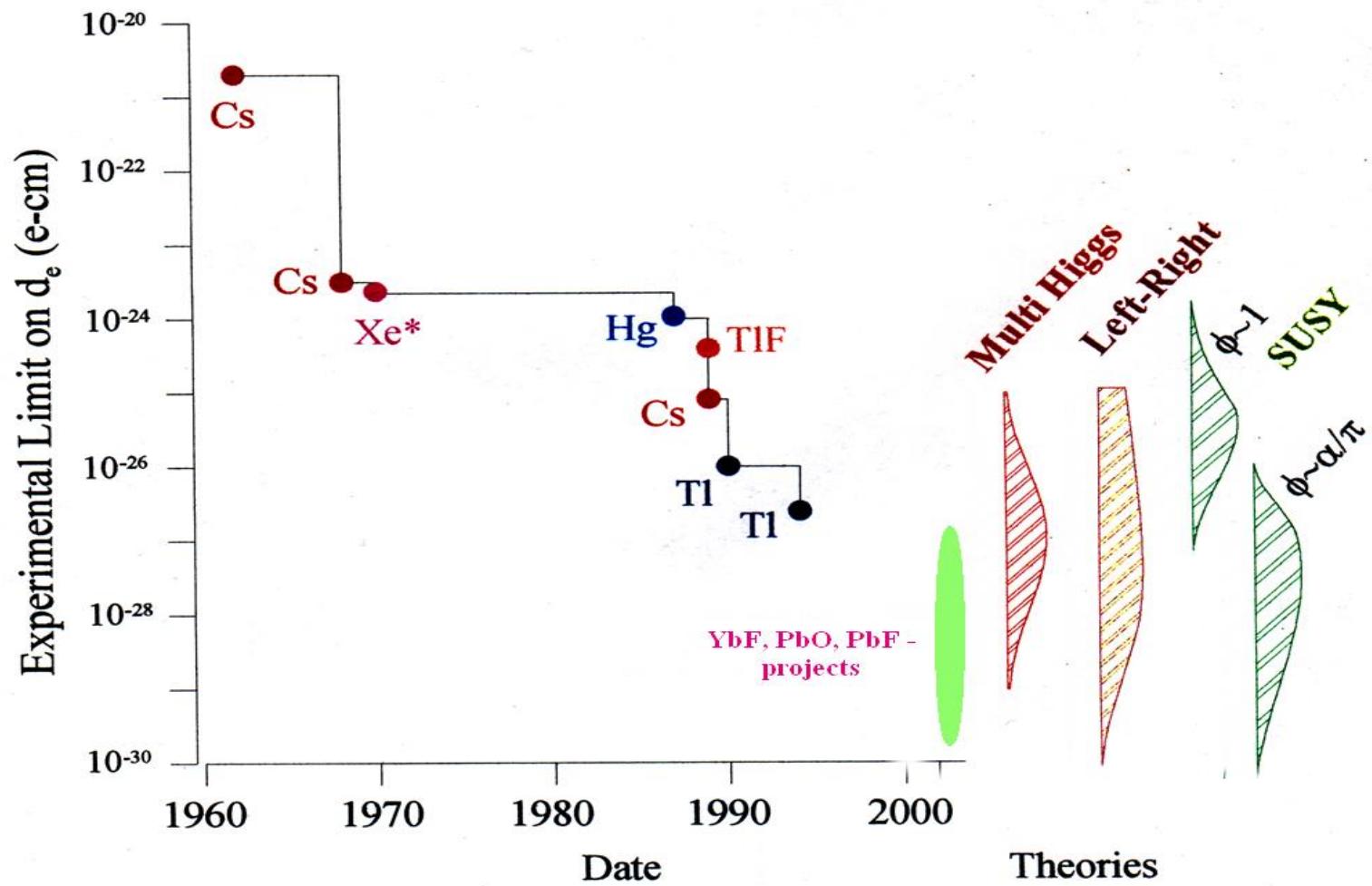
$$E_{int} \approx Z^2 \alpha^2 \frac{e}{a_0^2}$$

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B. Regan, E. Commins, C. Schmidt,
D. DeMille, PRL **88**, 071805 (2002)

Best experimental limits on d_e versus time:



Problems

- Choose of molecule
- Calculation of internal electric field.
- Producing of molecular radicals
- Cooling of molecules, population
- Deceleration of molecules, line width
- Quadratic Stark effect
- Polarization

Calculation of the enhancement factor in molecules

- YbF – **$8.3 \cdot 10^{-26} \text{ e}\cdot\text{cm}/\text{Hz}$**

Mosyagin N.S., Kozlov M.G., Titov A.V. J.Phys.B, 1998, v.31, L767.

Kozlov M.G. J.Phys.B, 1997, v.30, L607.

Titov A.V.,Mosyagin N.S.,Ezhov V.F. Phys. Rev. Lett. v.77,5346 (1996)

Kozlov M.G., Ezhov V.F. Phys Rev A v 49 4502(1994)

- PbO – **1.6**

Choose of molecule

Kozlov M.G., DeMille

T.A. Isaev, et. al. Phys

Calculation of internal electric field.

- PbF – **$7.1 \cdot 10^{-26} \text{ e}\cdot\text{cm}/\text{Hz}$**

Dmitriev Yu.Yu. et.al. Phys.Lett.A, 1992, v.167, p.280

- TlF – **$3.2 \cdot 10^{-20} \text{ e}\cdot\text{cm}/\text{Hz}$**

Petrov A.N., Mosyagin N.S., Isaev T.A., Titov A.V., Ezhov V.F. Phys.Rev.Lett., 2002, v.88, 073001.

EDMe enhancement in atoms and molecules

		TIF	YbF	PbF	PbO*	TI
Applied field (kV/cm)		15	25	15	$15 \cdot 10^{-3}$	100
Internal field E_{int} (GV/cm)			30	100	30	0.06
		Beam	Beam	Trap	Cell	Beam
Coherence time τ (ms)		3-30	3-30	>1000	0.1	3

Sensitivity:

$$\frac{\text{shift}}{\text{resolution}} \propto \frac{E}{(1/T)(S/N)^{-1}} = E \cdot T \cdot \sqrt{N}$$

Stability of magnetic field

For EDM equal to $10^{-27} \text{ e}\cdot\text{cm}$

PbF ($\text{X}^2\Pi_{1/2}$,
 $J=1/2$, $F=1$)

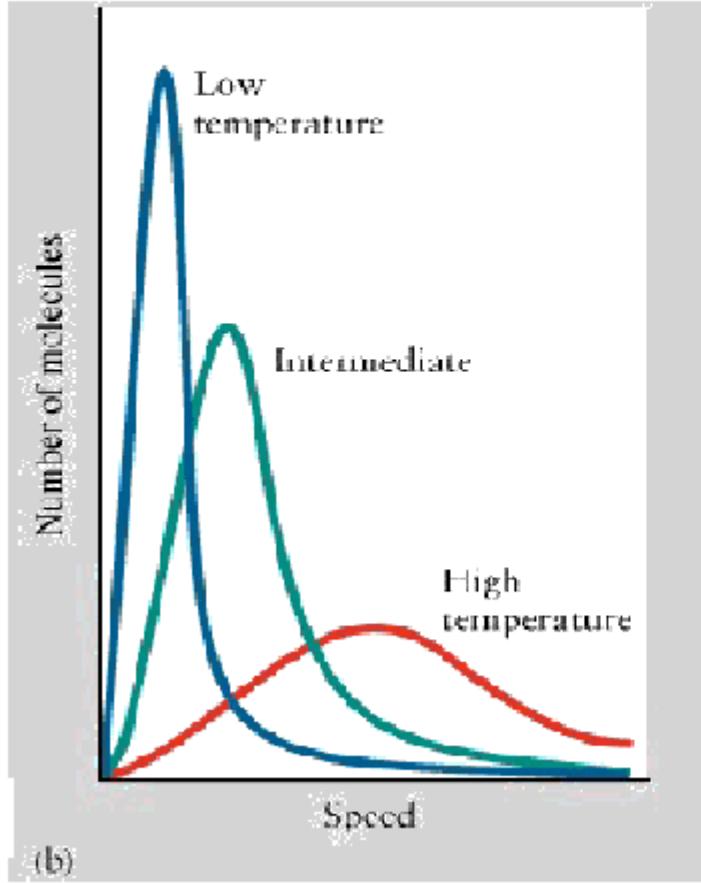
$5 \cdot 10^{-2} \text{ Hz}$

8.8 pT

n $1.6 \cdot 10^{-7} \text{ Hz}$ 0.005 pT

$$\frac{\text{shift}}{\text{resolution}} \propto \frac{E}{(1/T)(S/N)^{-1}} = E \cdot T \cdot \sqrt{N}$$

Maxwell distributions



- For a given molecule, as T increases, the average speed and the range of speeds increase

$$v \propto \sqrt{\frac{\text{temperature}}{\text{molar mass}}}$$

$$T_{\text{vap}} \sim 1000 \text{ K}$$

$$v \sim 300 \text{ m/s}$$

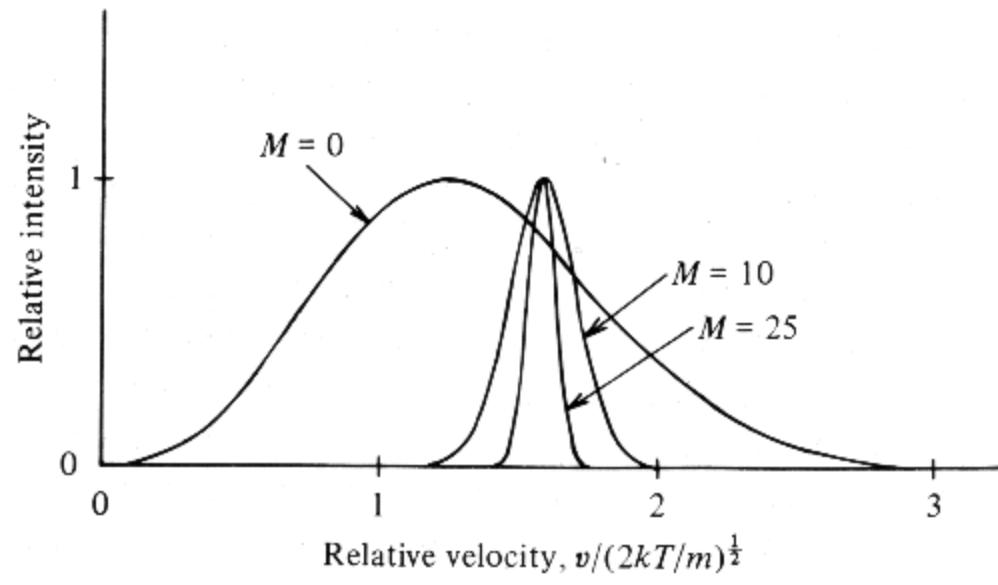
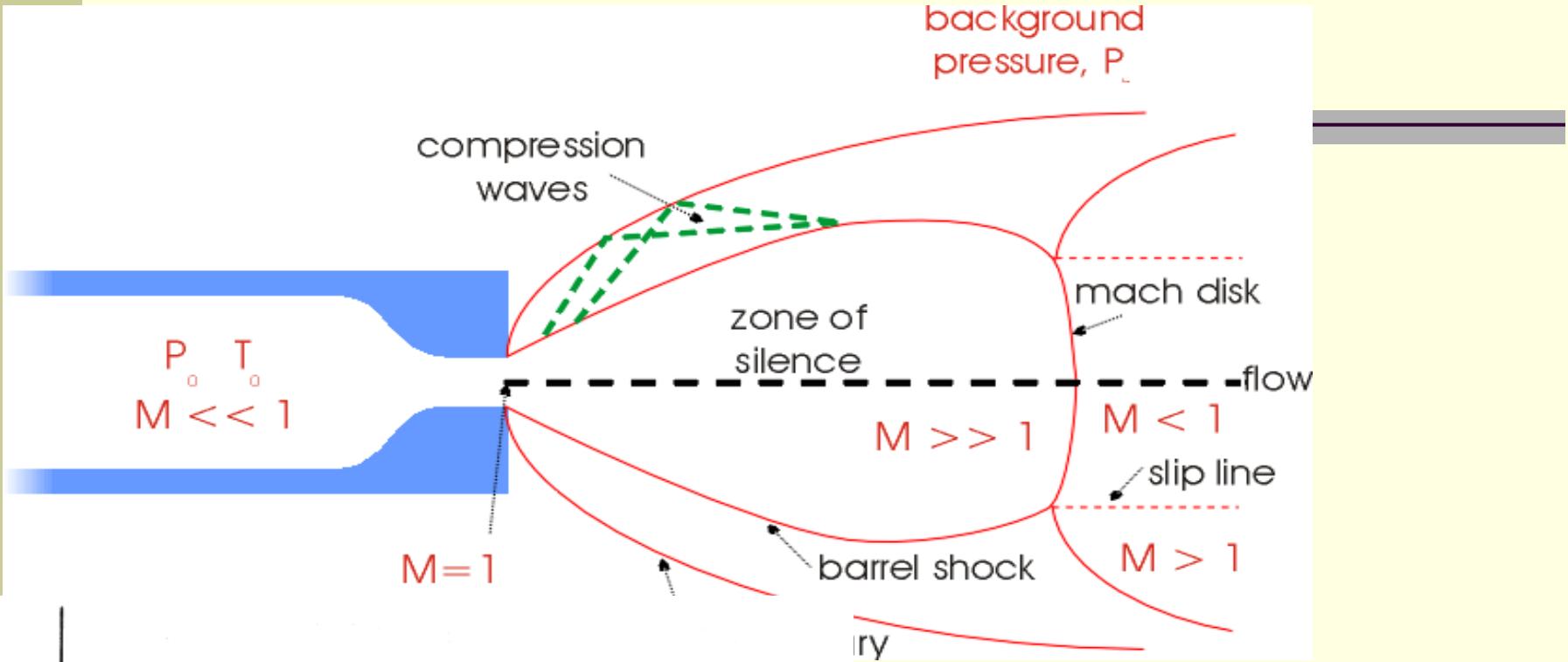
$$T_{\text{rot}} < 1 \text{ K}$$

$$N_{\text{hf}} \sim 10$$

$$\Omega \sim 10^{-5}$$

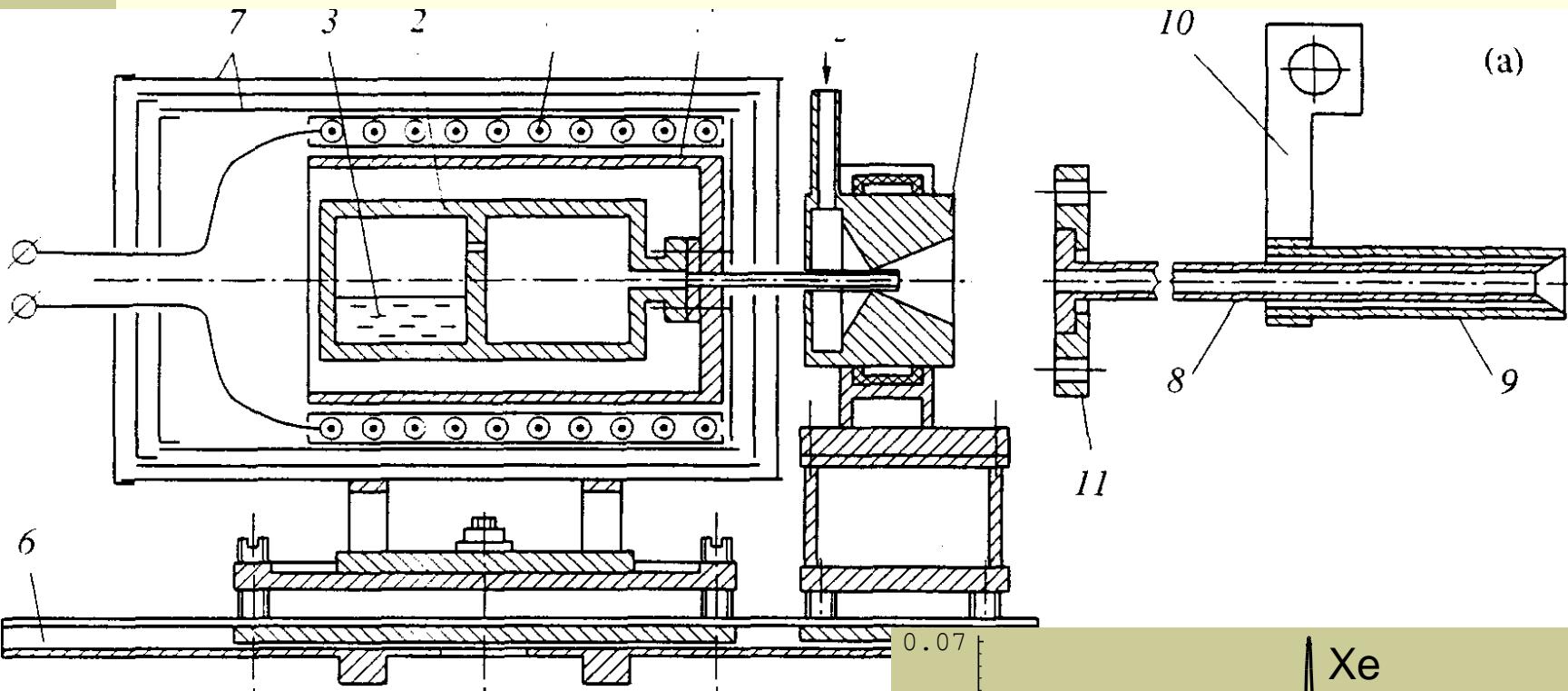
$10^{-9} !!!$

Supersonic expansion.



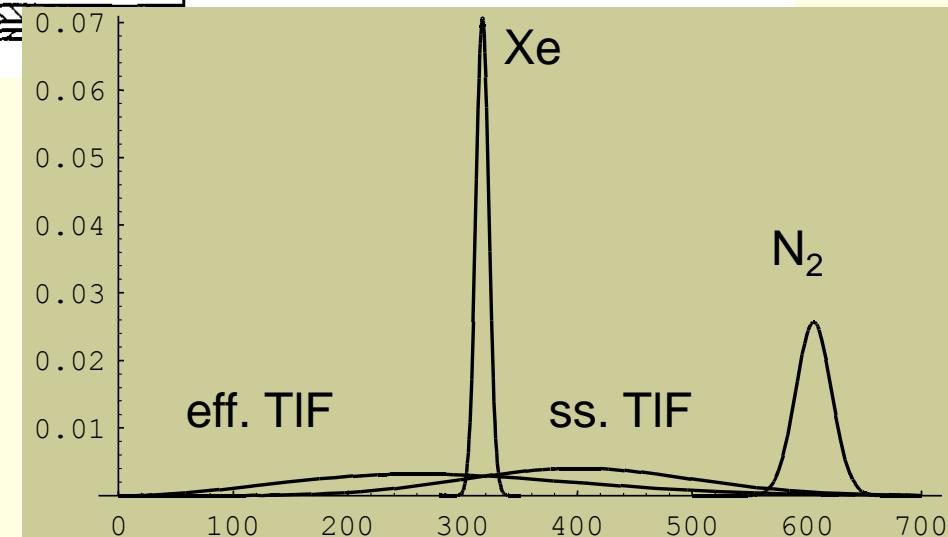
$$x_M = \frac{2}{3}(d) \left(\frac{P_0}{P_b} \right)^{\frac{1}{2}}$$

PNPI molecular beam generator

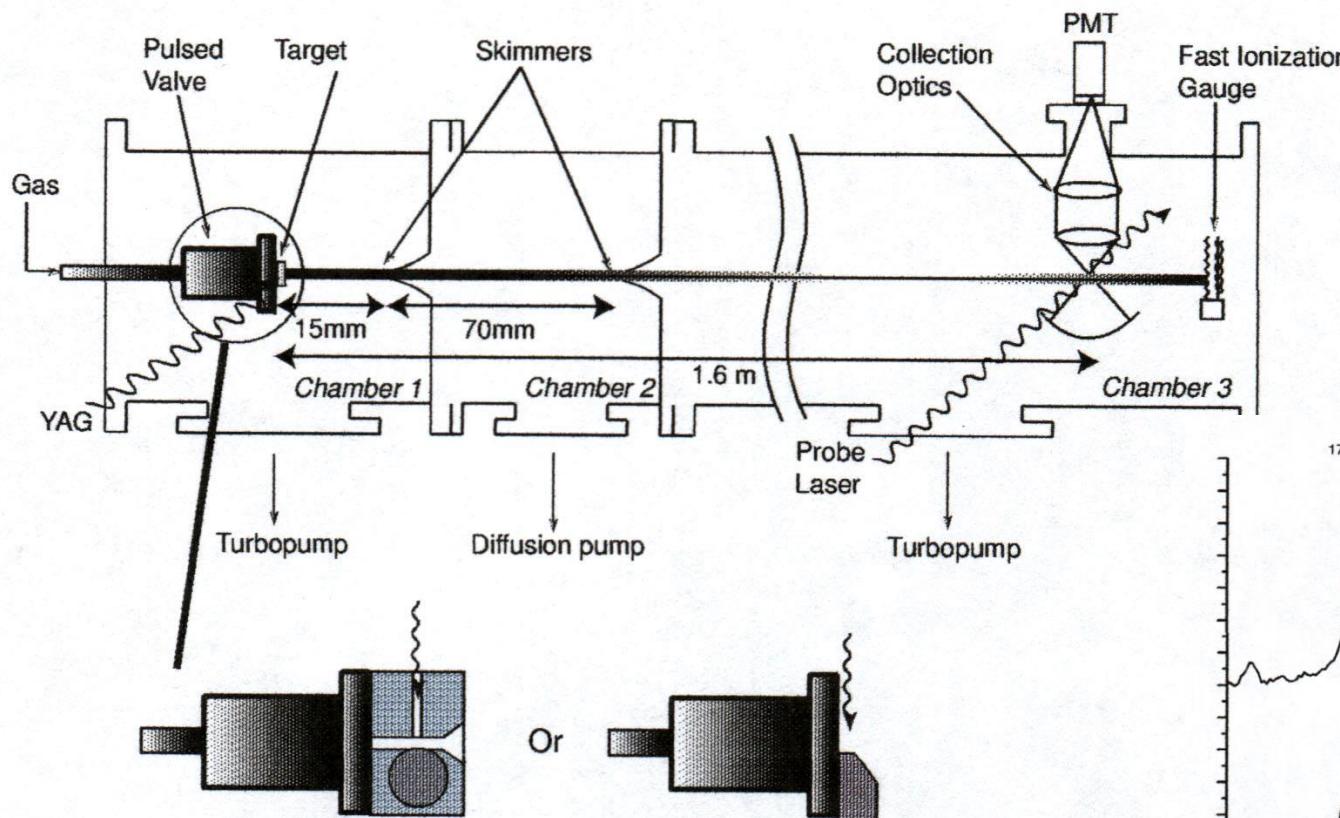


(a)

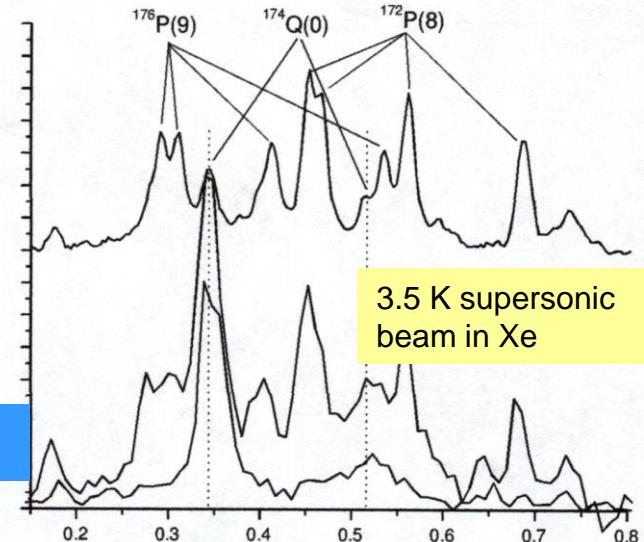
Unfortunately using of Xe as carrier gas
too is too expensive
Producing of molecular radicals !!!



YbF molecular beam source (Imperial College UK, PNPI)



effusion beam at 1500 K



J Phys B, 35, (2002) 5013,

A – X spectrum of YbF near the 542811 GHz.

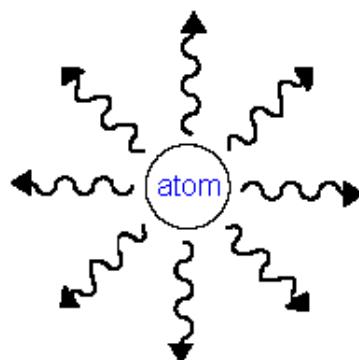
Optical cooling and deceleration of atoms

Absorption of N photons



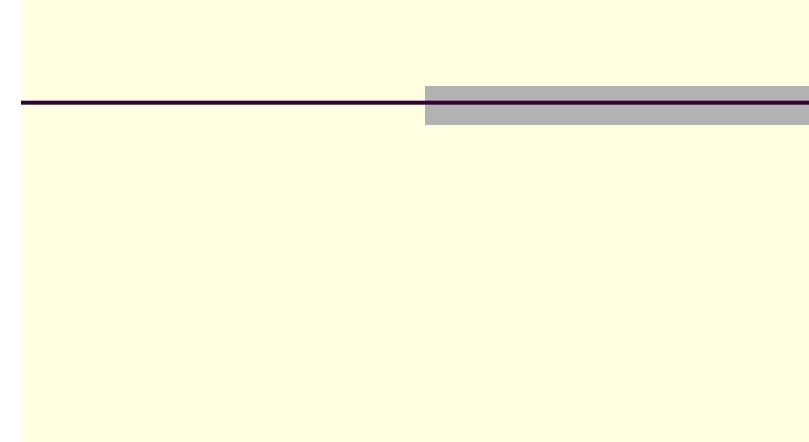
$$\Delta \vec{p} = N \hbar k \hat{z}$$

Emission of N photons

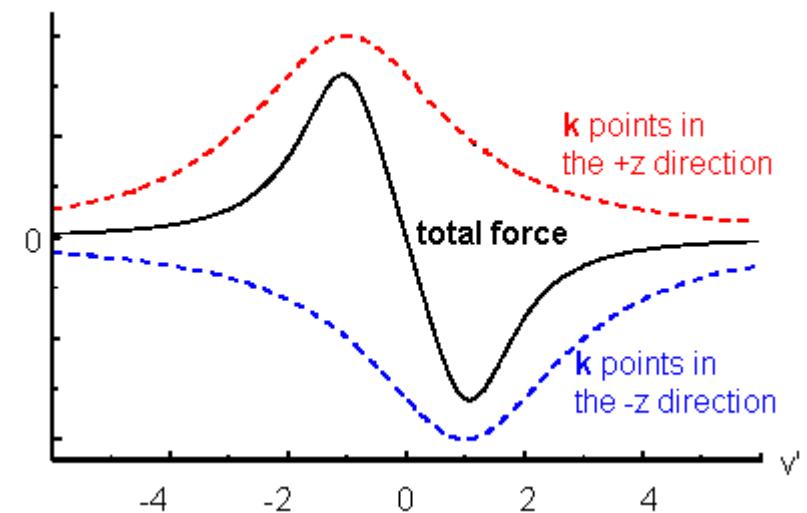


$$\langle \Delta \vec{p} \rangle = 0$$

$$\omega = \omega_0 \left(1 \pm \frac{v_a}{c} \right)$$



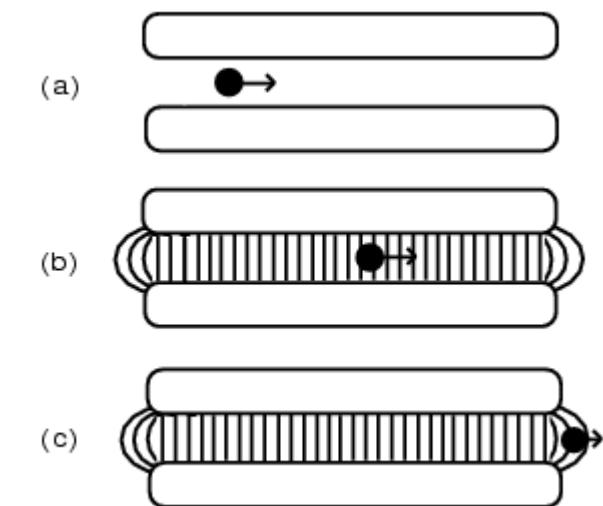
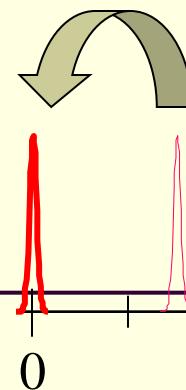
Force



The acceleration of the atom depends on the number of photons per second absorbed and then re-emitted via spontaneous emission.
The maximum value of the acceleration is $a_{\max} = \hbar k \Gamma / (2m)$

$$\Delta\omega \cdot \Delta t \approx h$$

$$\rightarrow \Delta\omega \rightarrow 0 \text{ & } \Delta t \rightarrow \infty$$



DECELERATION OF LIGHTER, LESS ENERGETIC MOLECULES HAS BEEN ACHIEVED WITH A TRAVELING POTENTIAL WELL

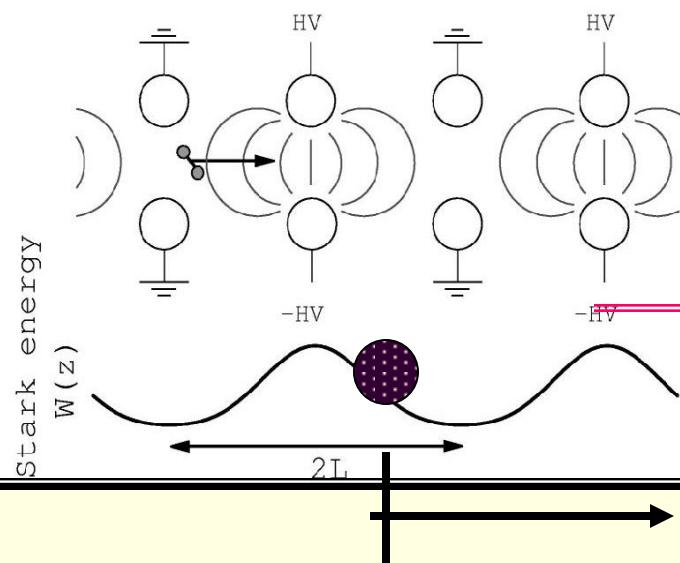


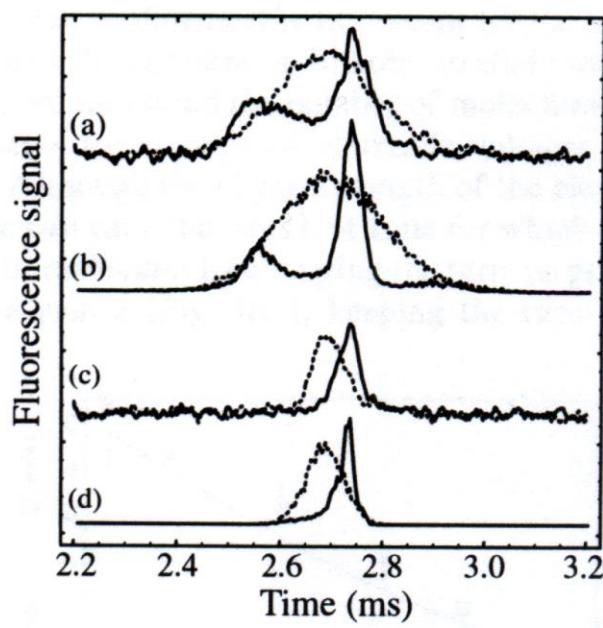
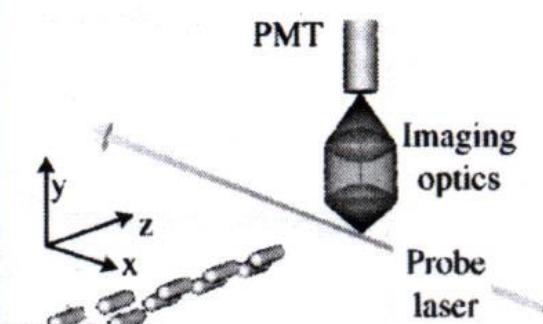
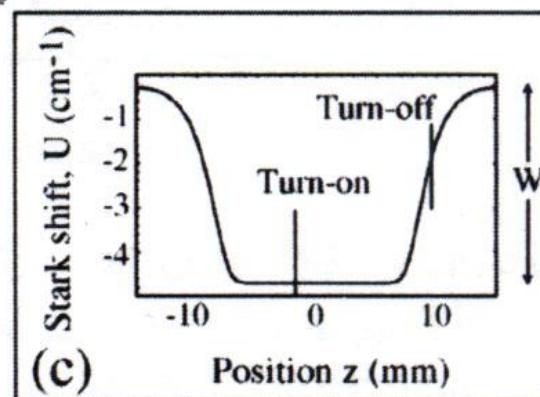
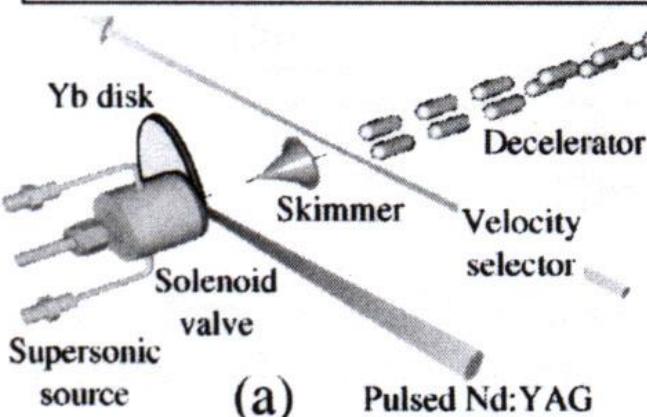
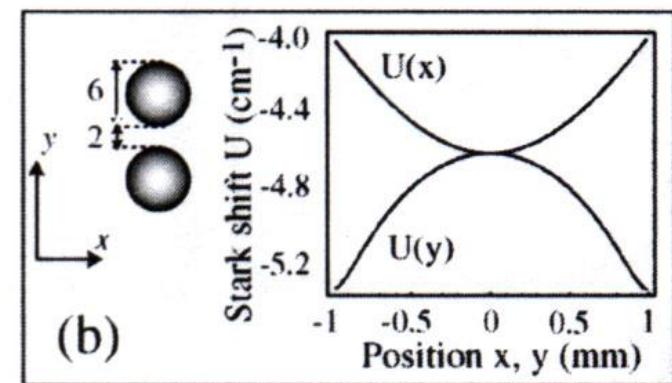
Figure from Meijer and coworkers,
PRL 84, 5744 (2000.)

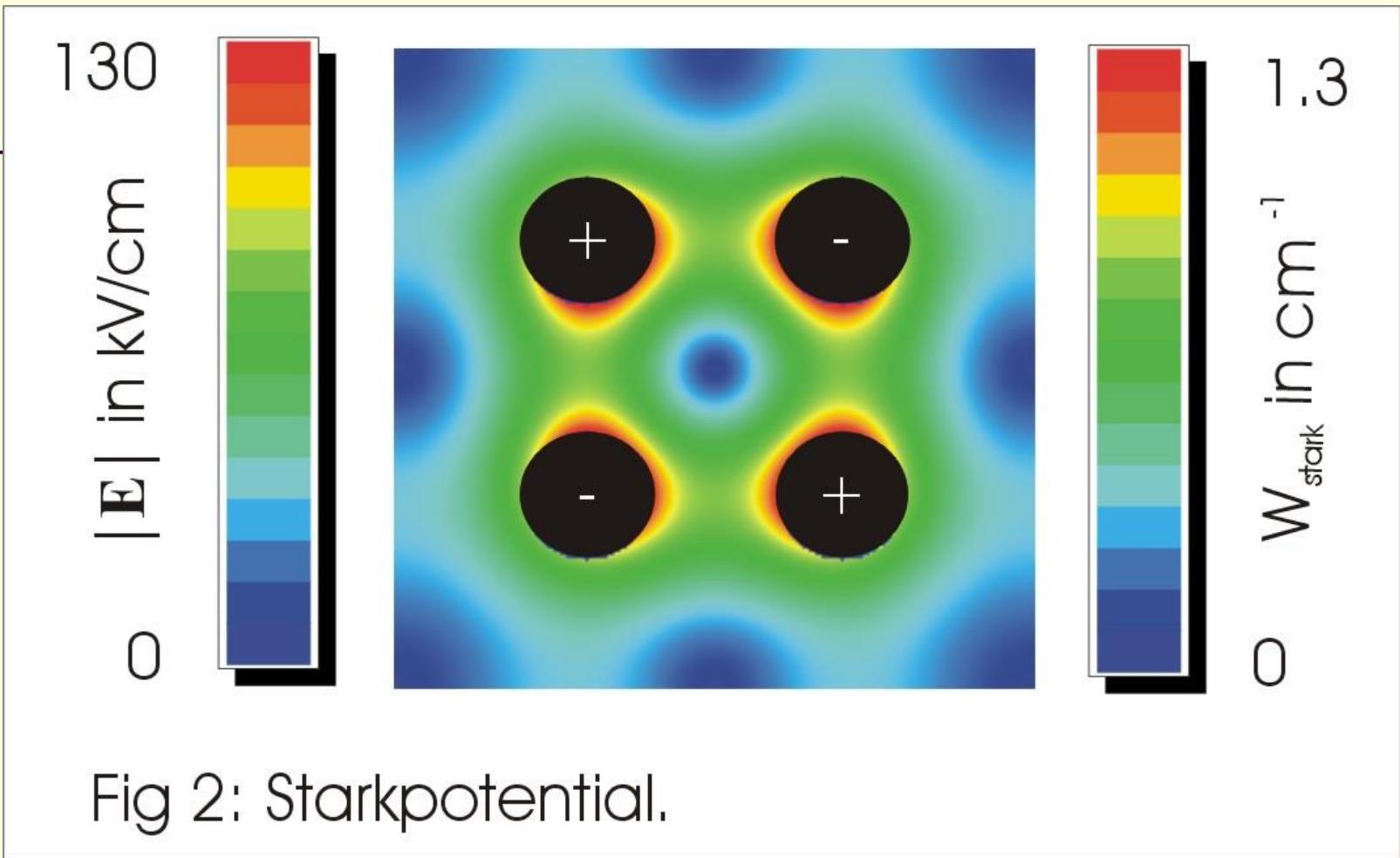
*Timed pulse sequence gives
decelerating trap potential.*

It was done for light molecules – CO !!!

YbF decelerator

(Imperial College UK, PNPI)





$$F = - \frac{\partial W}{\partial r}$$

fast
molecules

molecules@RT

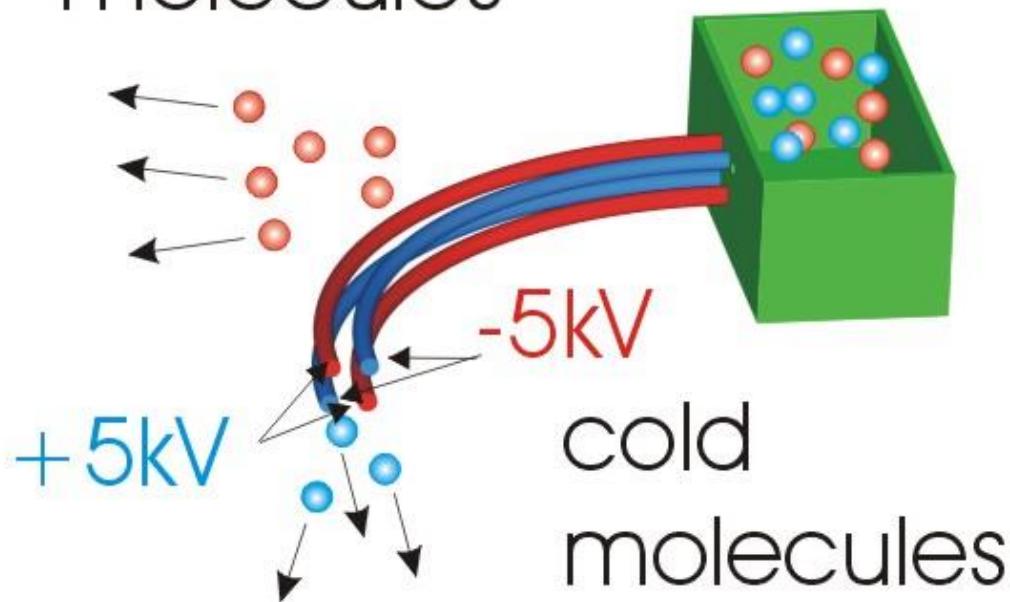
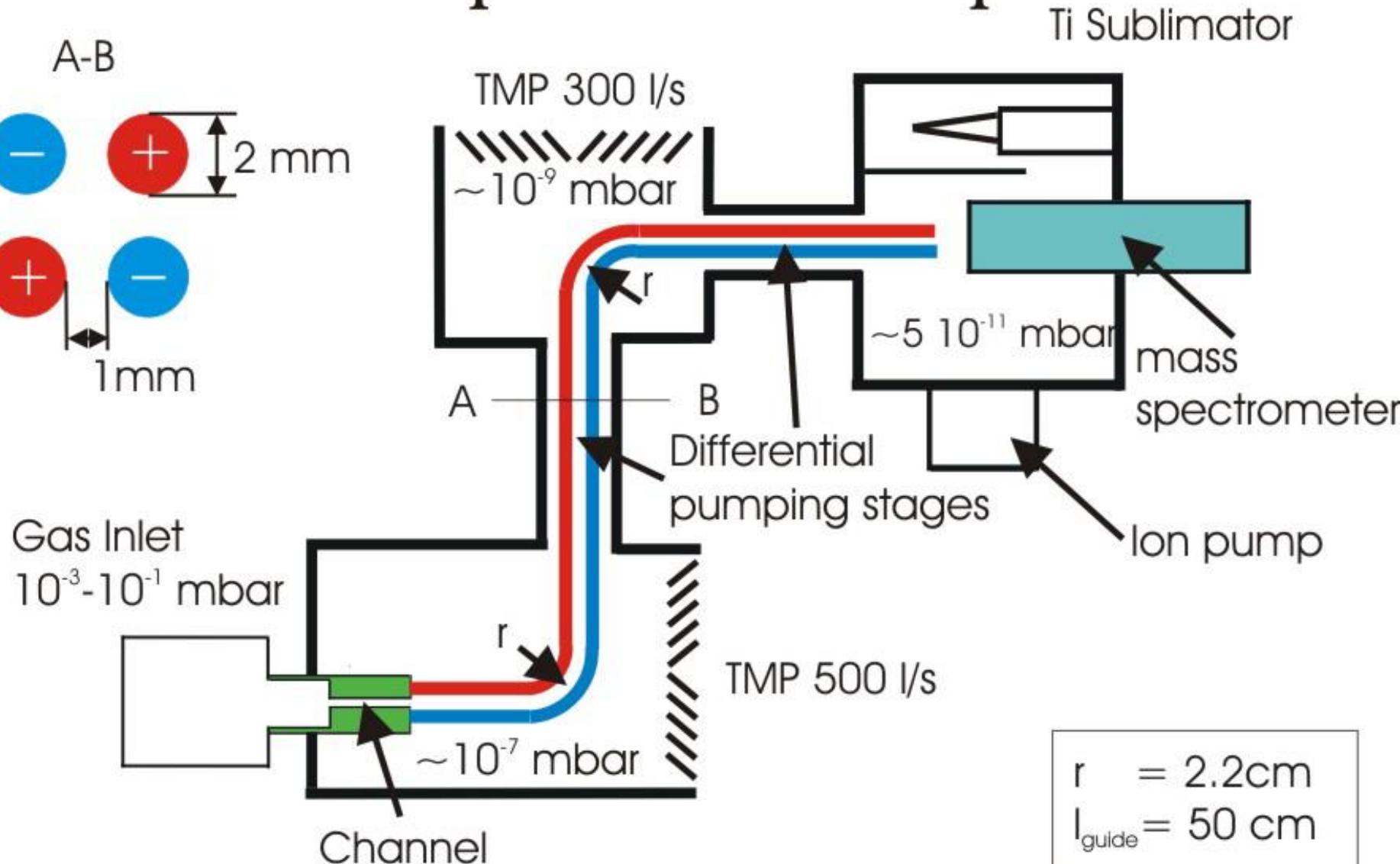
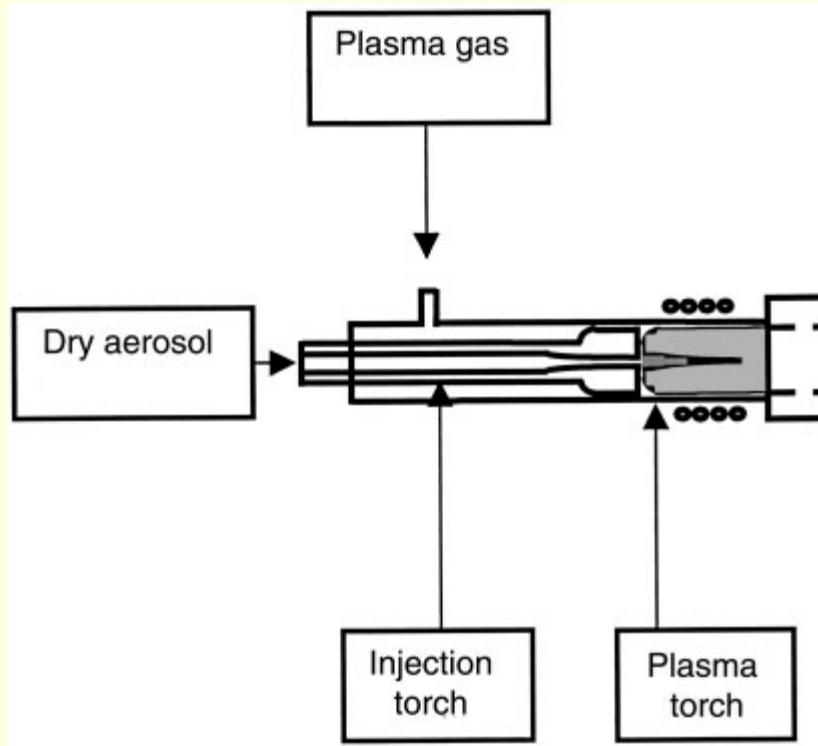


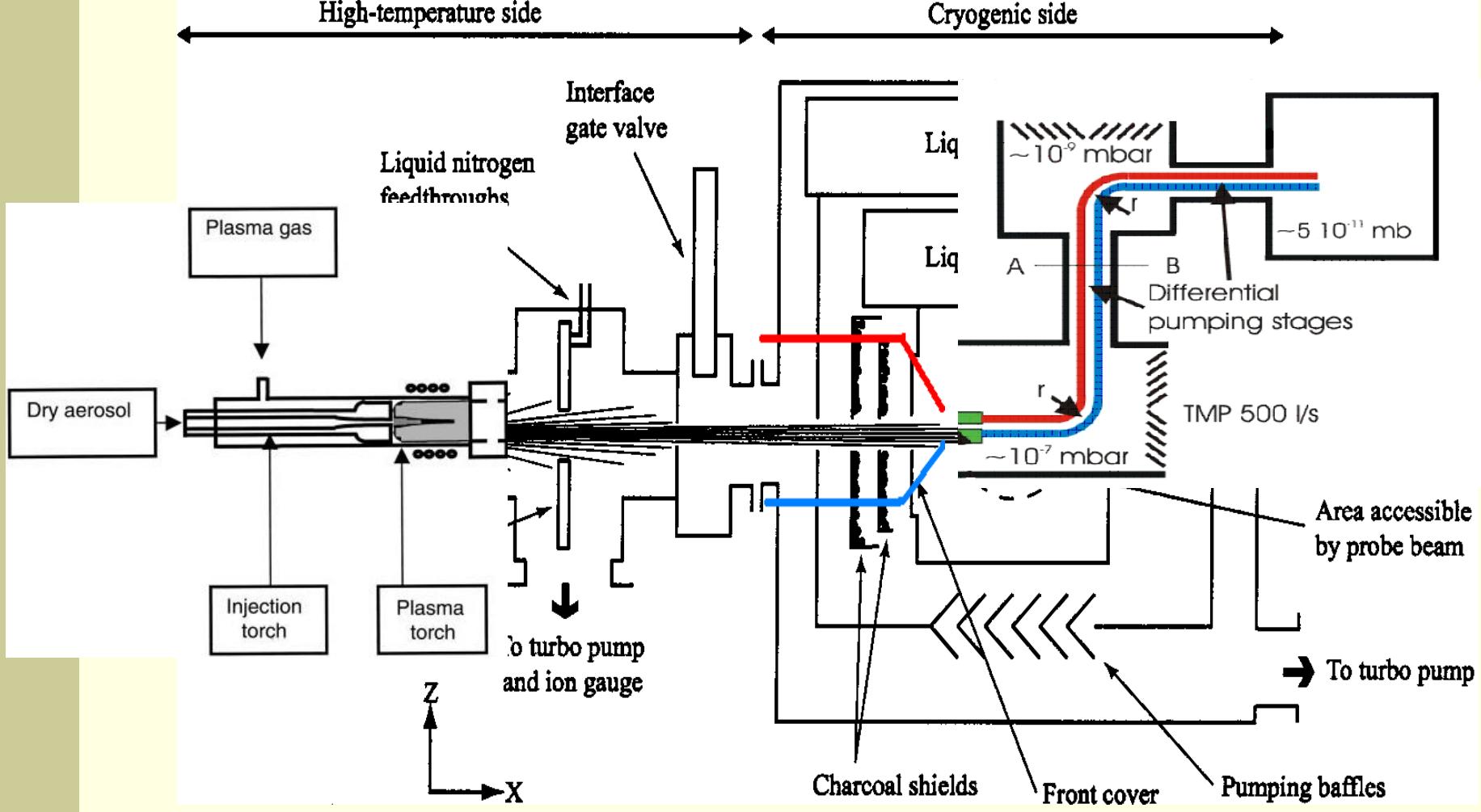
Fig. 3: Basic Stetup.

Experimental Setup









Schematic view of the apparatus. Diameter of the oven orifice is 3.9 mm.
 Diameter of the orifice in the cell front cover was varied between 0 and 3 mm.
 Distance between oven orifice and cell orifice is 25 cm.

Measuring the Electron's Electric Dipole Moment using trapped PbF molecules

The University of Oklahoma

Neil Shafer-Ray

Kim Milton

Eric Abraham

George Kalbfleisch

University of Latvia

Marcis Auzinsch

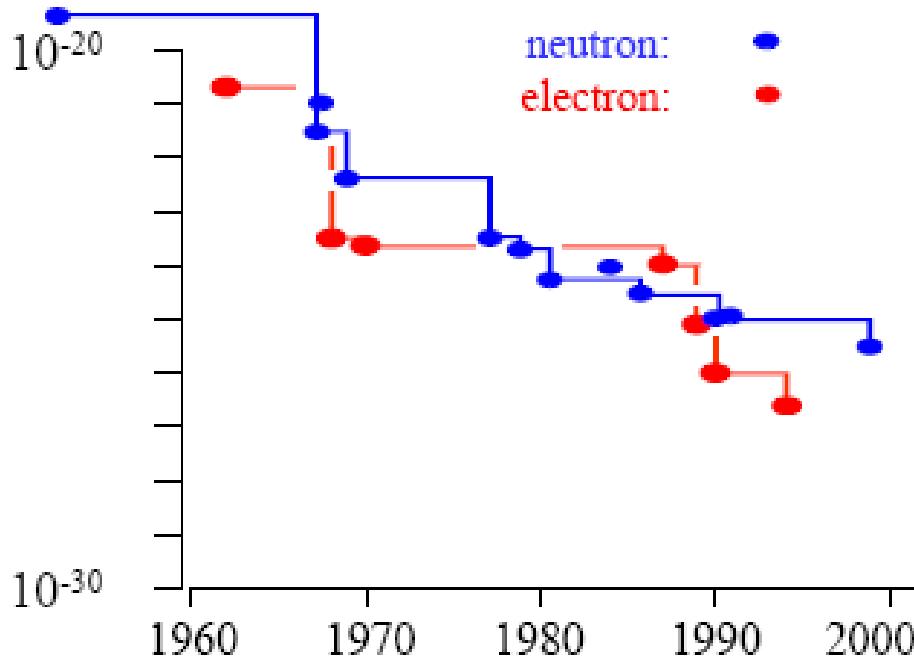
St. Petersburg Nuclear Physics Institute

Victor Ezhov

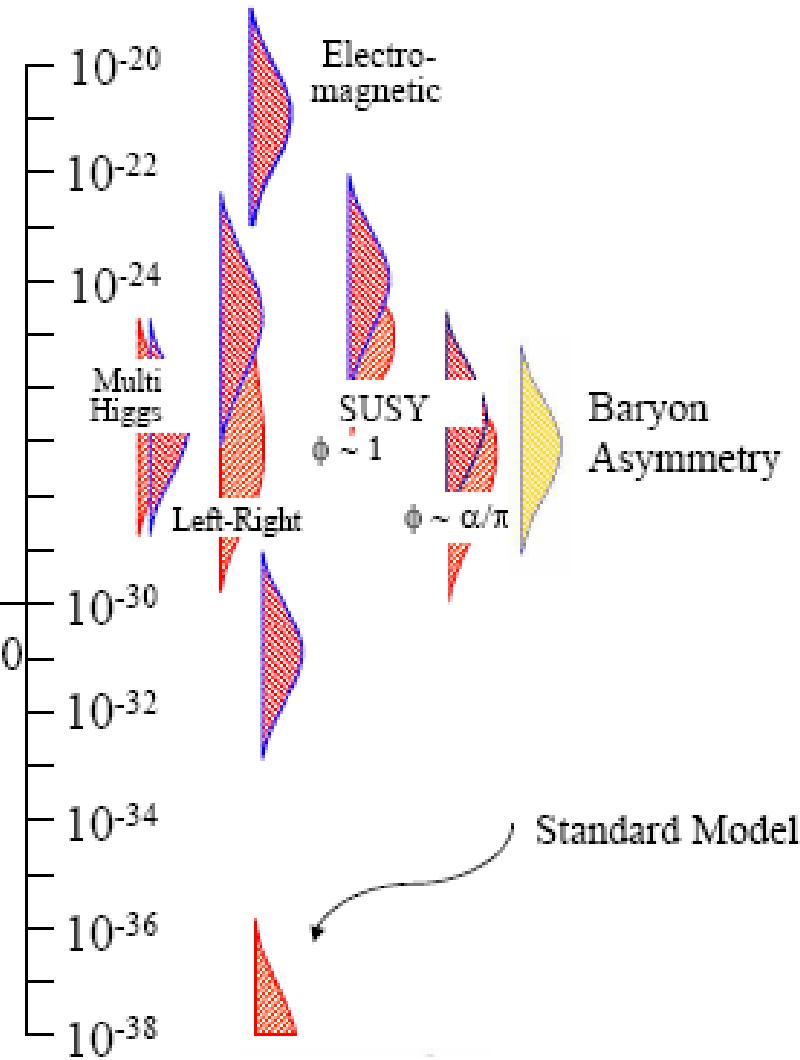
Mikhail Kozlov

EDM limits: the first 50 years

Experimental Limit on d (e cm)



neutron:
electron:



Updated from Barr: Int. J. Mod Phys. A8 208 (1993)

Макроскопические проявления квантовомеханических явлений

One dimensional Harmonic oscillator Hamiltonian for magnetic moment in the gradient of magnetic field

$$-\frac{\hbar}{m} \frac{d^2 u}{dx^2} + \left(\overrightarrow{\mu H}_W \frac{x^2}{\rho^2} - E \right) u = 0$$

Energy of Quantum levels

$$E_n = \hbar \sqrt{\frac{2|\overrightarrow{\mu H}_W|}{m\rho^2}} \left(n + \frac{1}{2} \right) = \frac{\lambda \hbar^2}{m} \left(n + \frac{1}{2} \right) \quad n = 0, 1, 2, \dots$$

For the gradient $\sim 2 \text{ T/mm}$ $\rightarrow E_n = 5.16 \cdot 10^{-31} \text{ Joule}$

The value of transversal velocity of neutron that equal to distances between energy levels must be about:

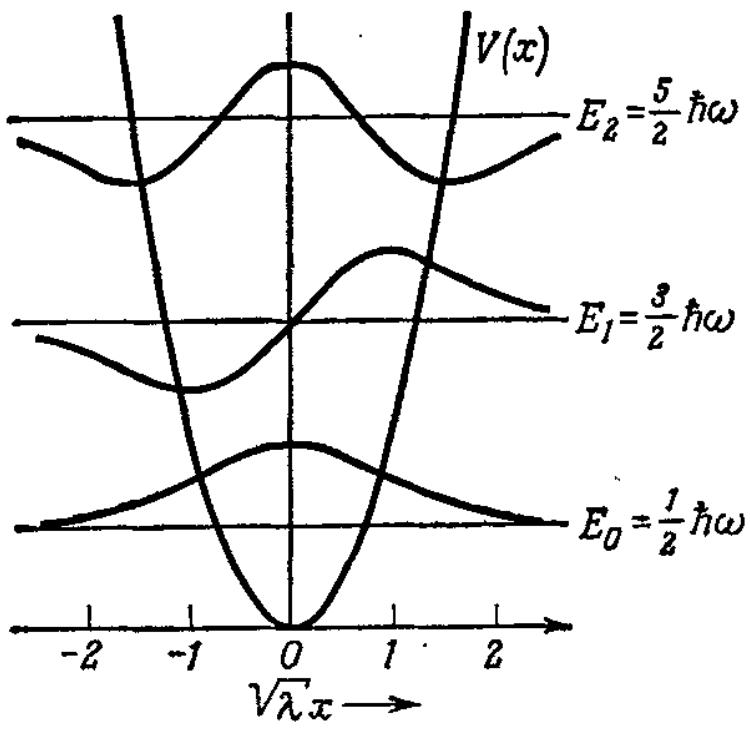
$V_{\text{trans}} = 0.0248 \text{ m/s}$

In case of two dimensional potential well

$$V = \frac{m}{2} \omega^2 (x^2 + y^2)$$

We receive two sets of quantum levels

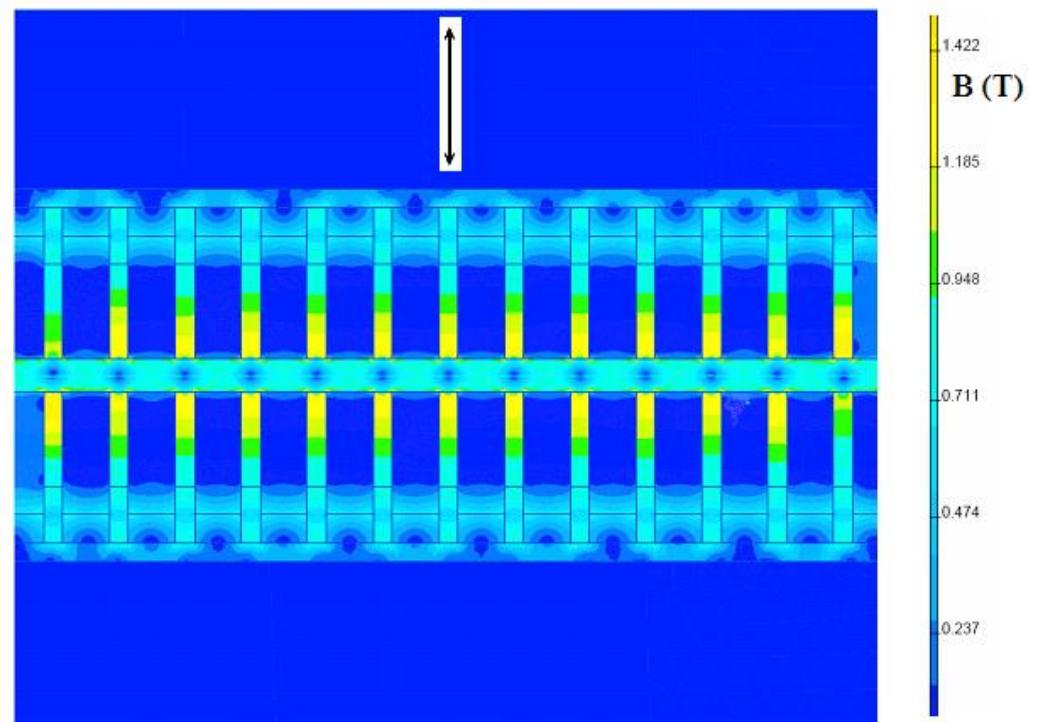
$$E = \hbar \omega_1 \left(n_1 + \frac{1}{2} \right) + \hbar \omega_2 \left(n_2 + \frac{1}{2} \right) \quad n_1, n_2 = 0, 1, 2, \dots$$



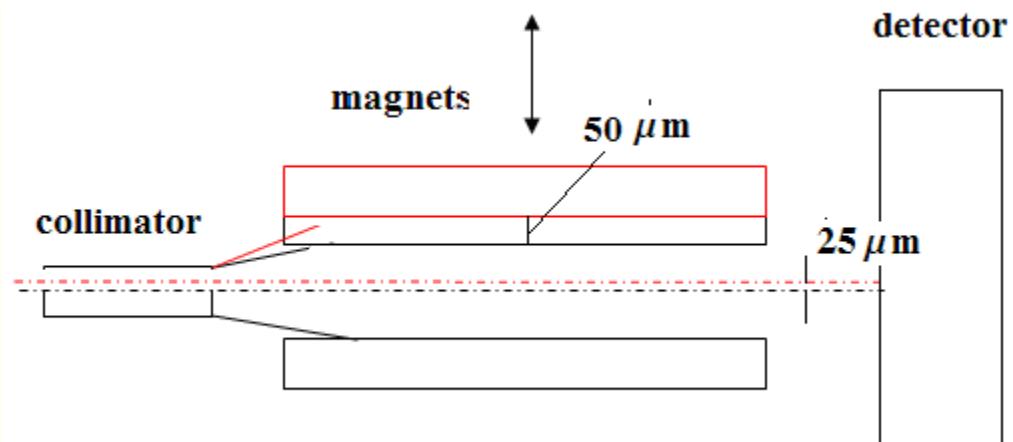
Фиг. 19. Потенциал, энергетические уровни и собственные функции гармонического осциллятора.

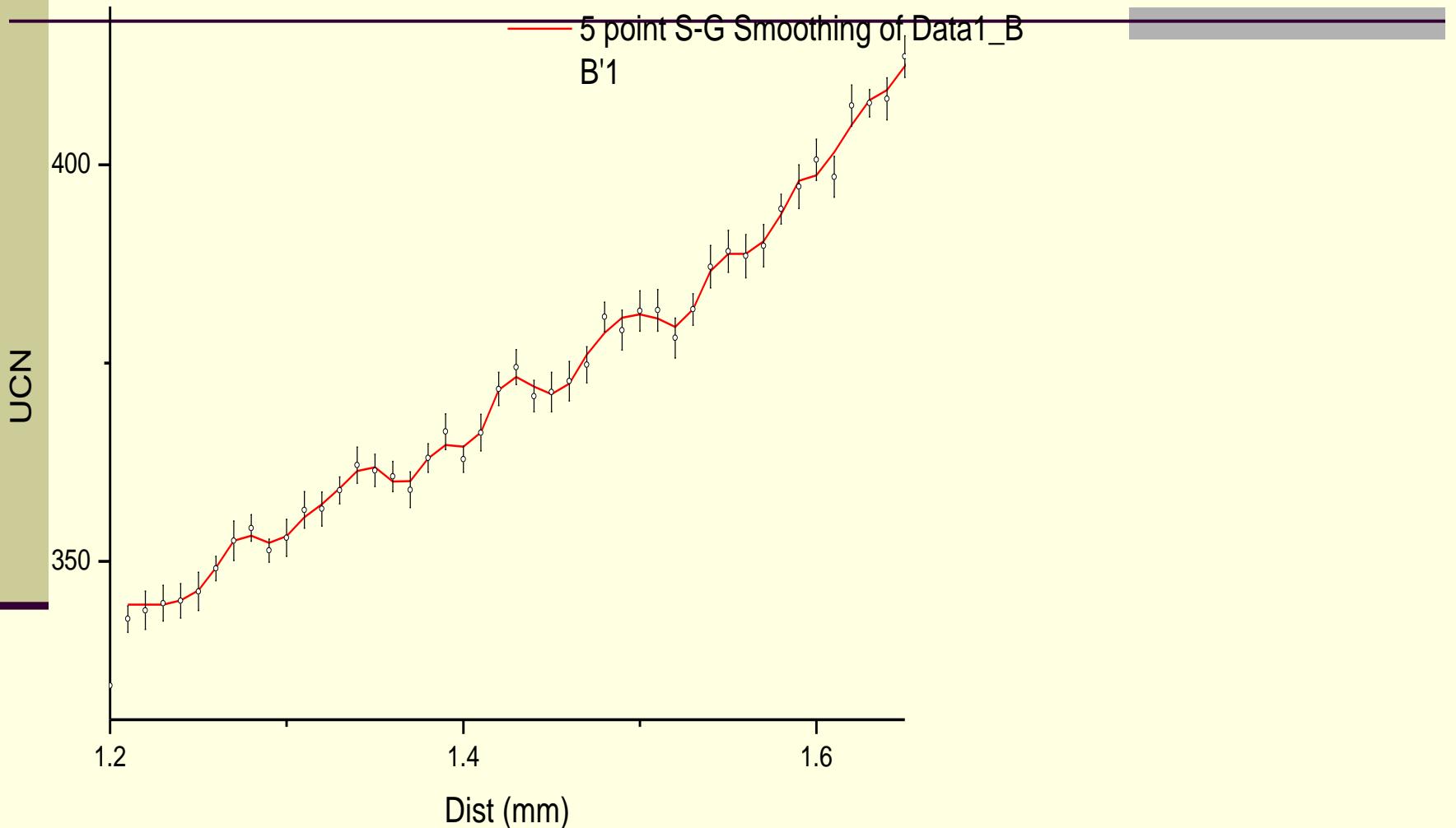
$$E = 5.16 \cdot 10^{-31} \text{ Joule} = 3.22 \text{ peV} = 0.00322 \text{ neV} = 5.33 \cdot 10^{-5} \text{ T}$$

Main experimental features of two-dimensional potential well

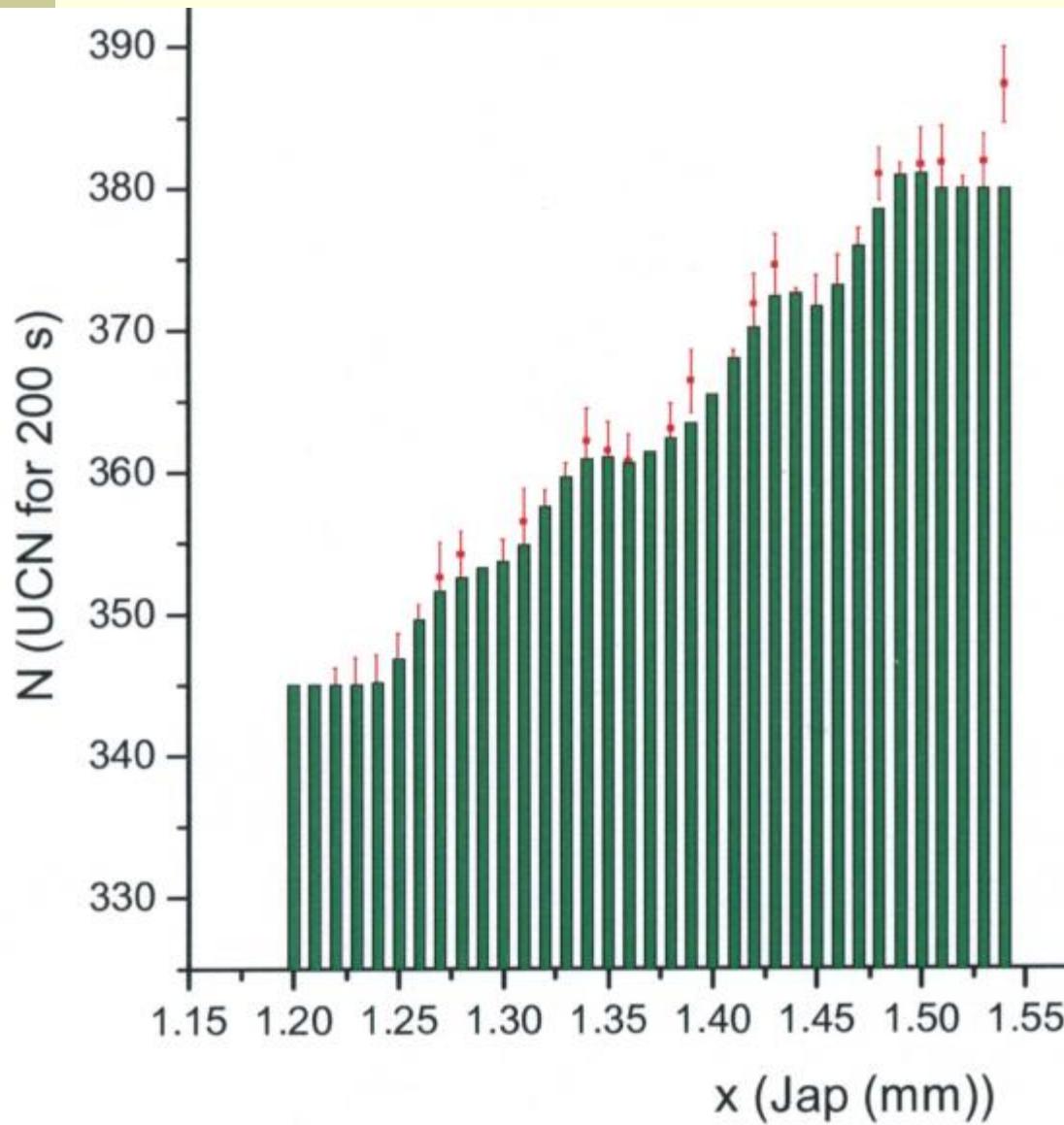


Potential well will be elliptical





Typical set of experimental data



To exclude the linear growth of background the differentiation of experimental data is used.

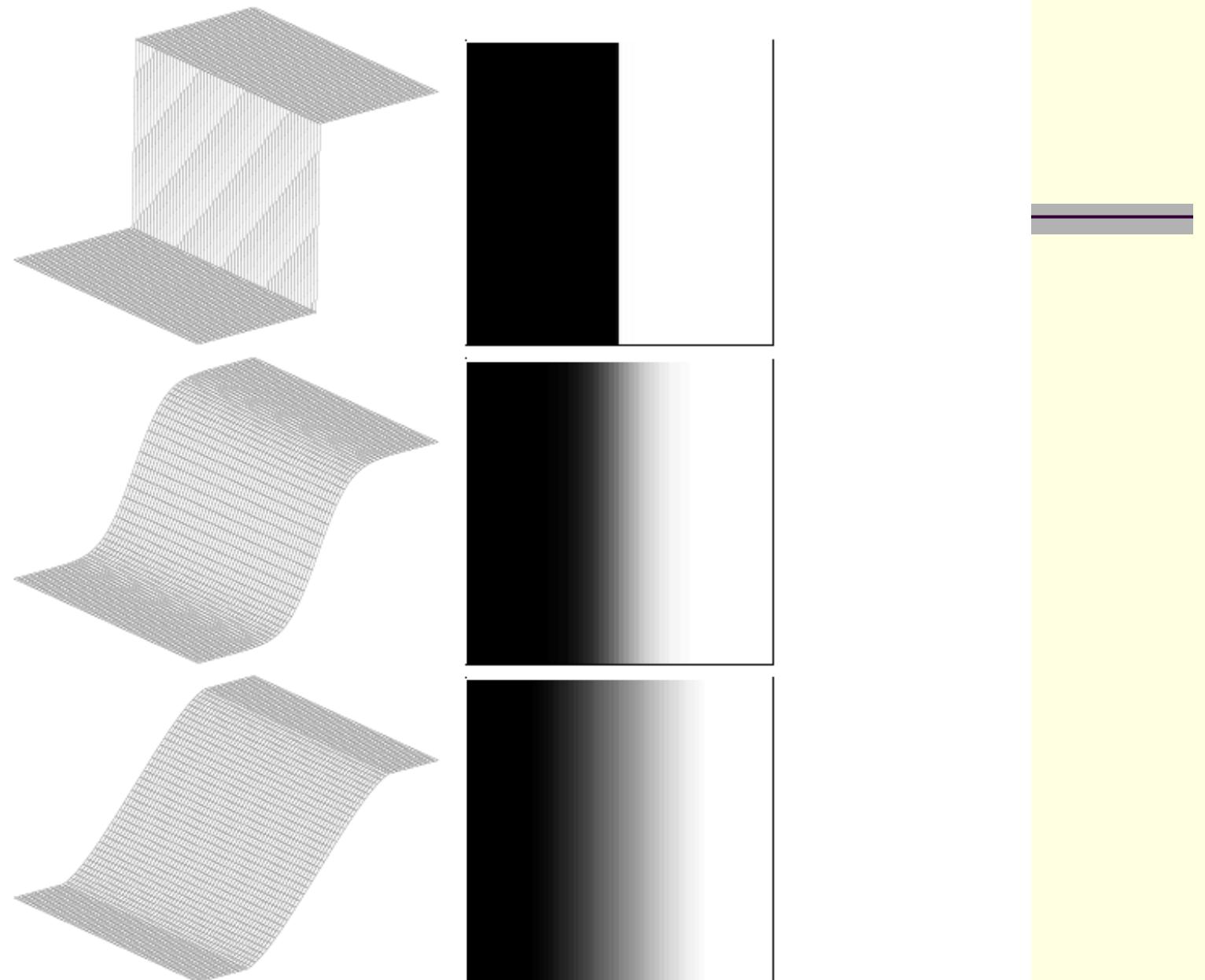
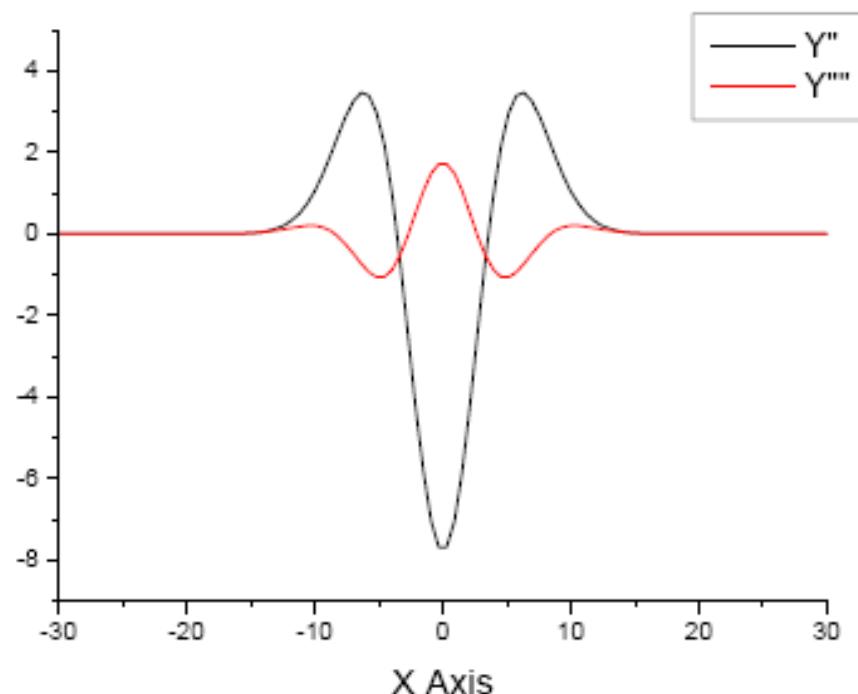
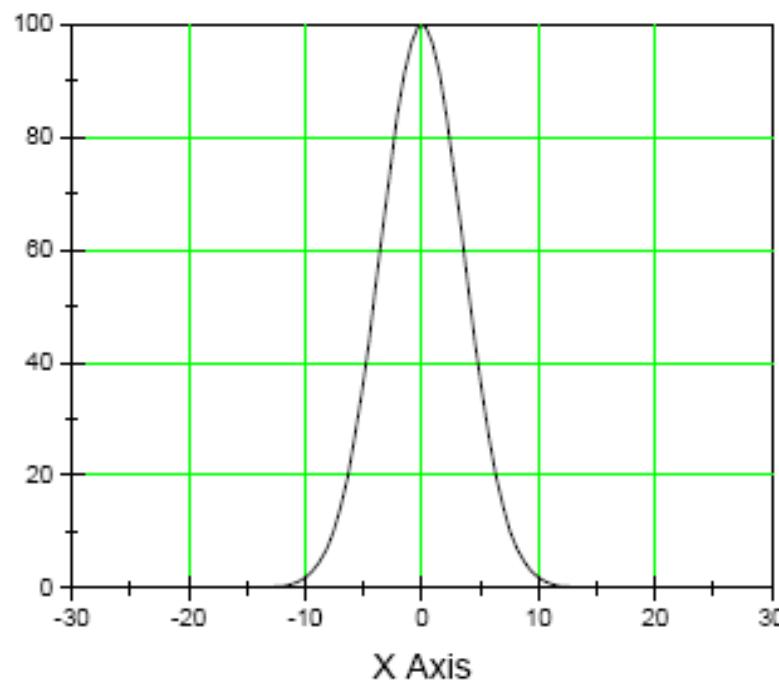


Figure 3: Intensity graphs (left) and images (right) of a vertical step function (top), and of the same step function smoothed with a Gaussian (middle), and with a pillbox function (bottom). Gaussian and pillbox have the same support and the same integral.

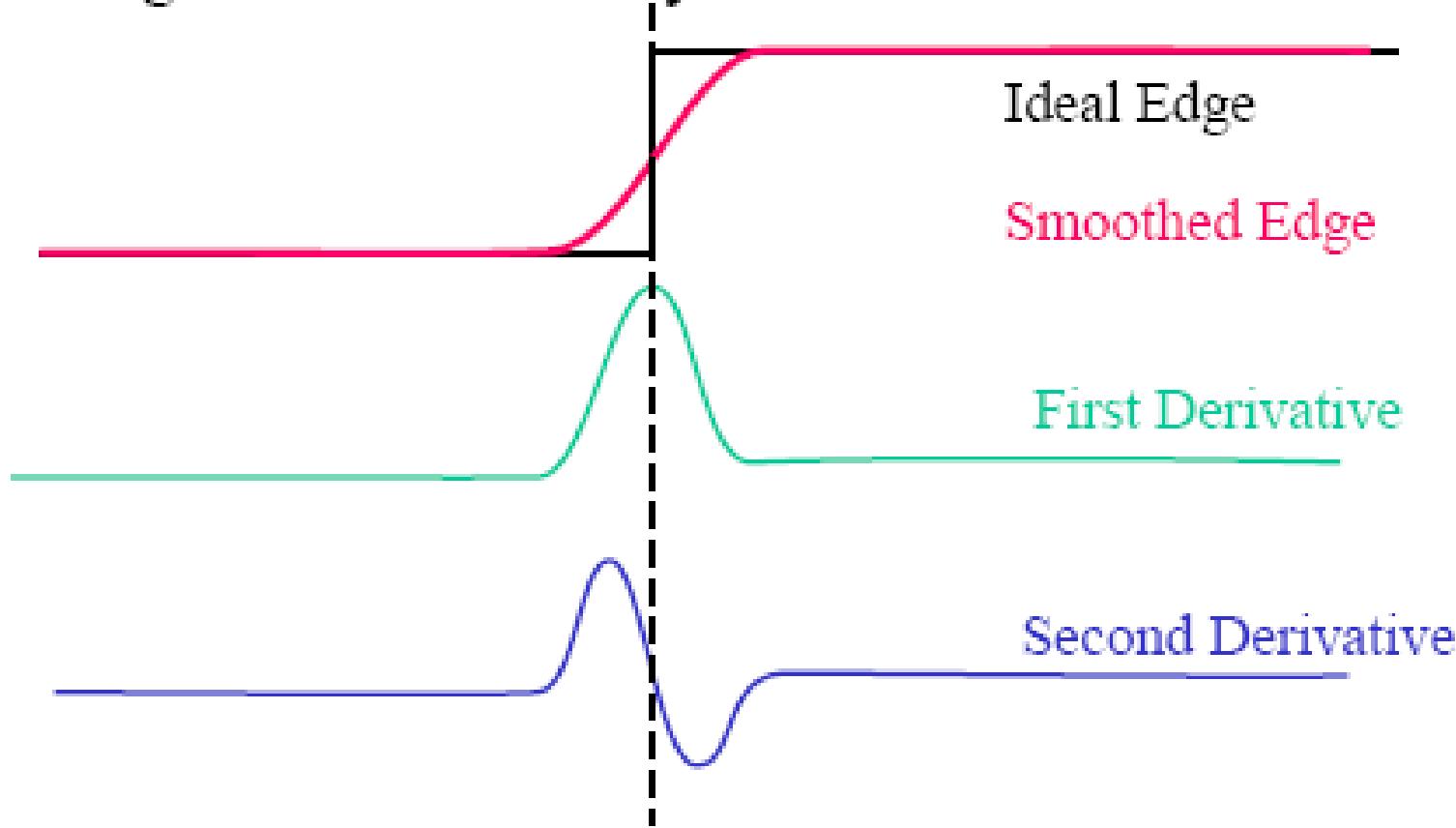
Использование производных четного порядка для разрешения пиков

- Амплитуда n-ой производной (четной) пика пропорциональна амплитуде пика и обратно пропорциональна n-ой степени полуширины
 - $Y(0)^{(n)} \sim Y(0)/(W^n)$

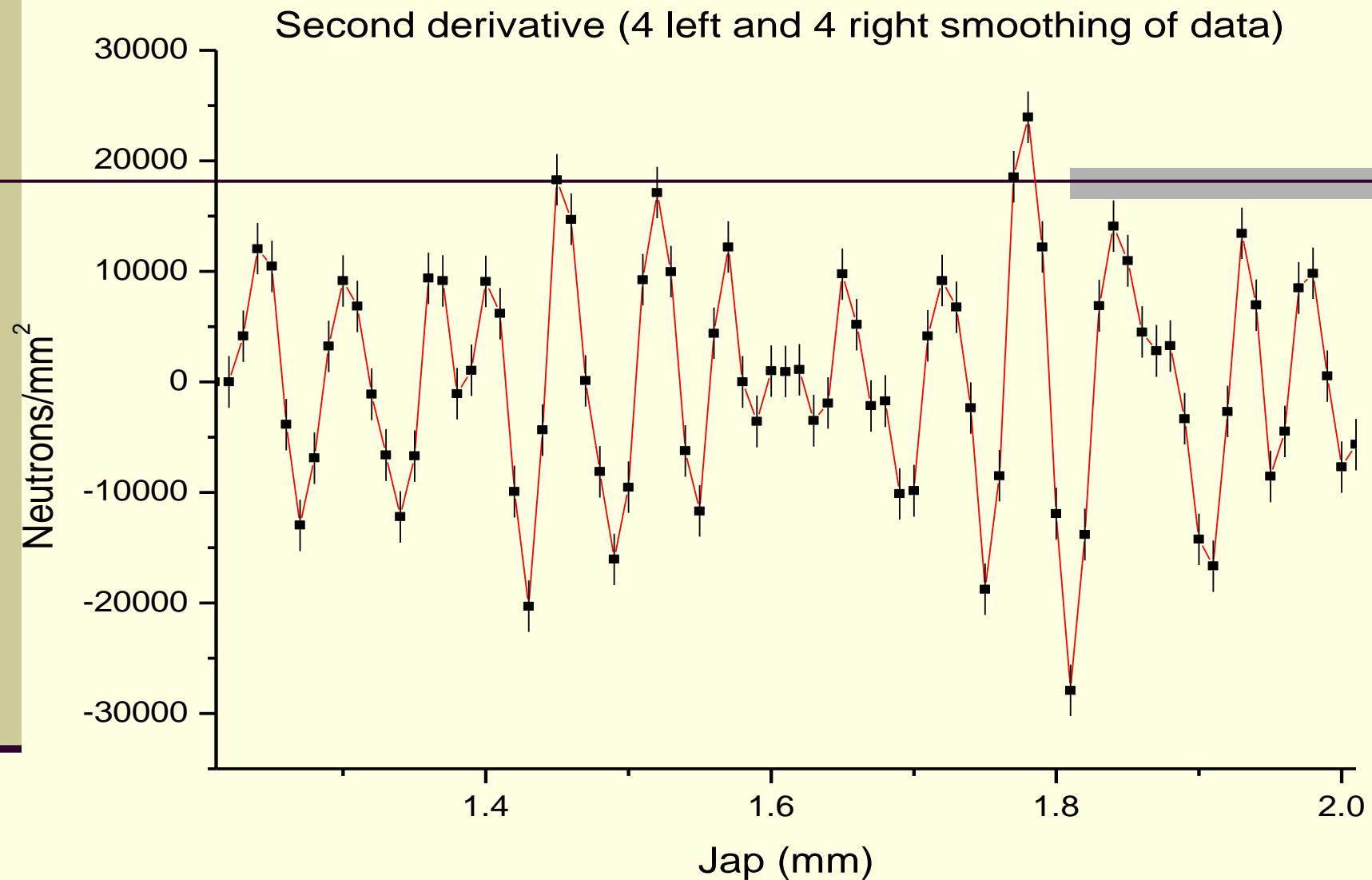


Edge is Where Change Occurs: 1-D

- Change is measured by derivative in 1D

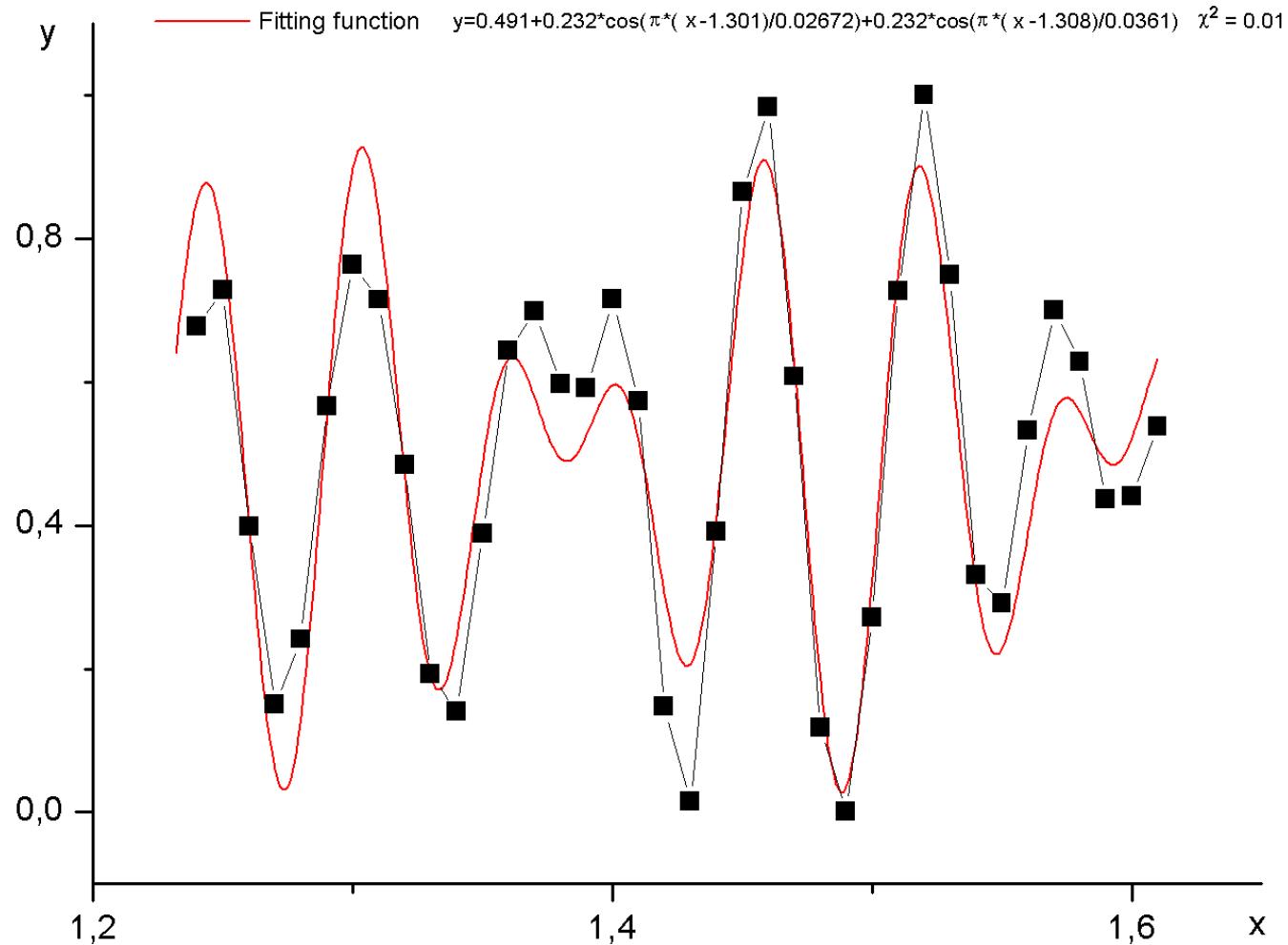


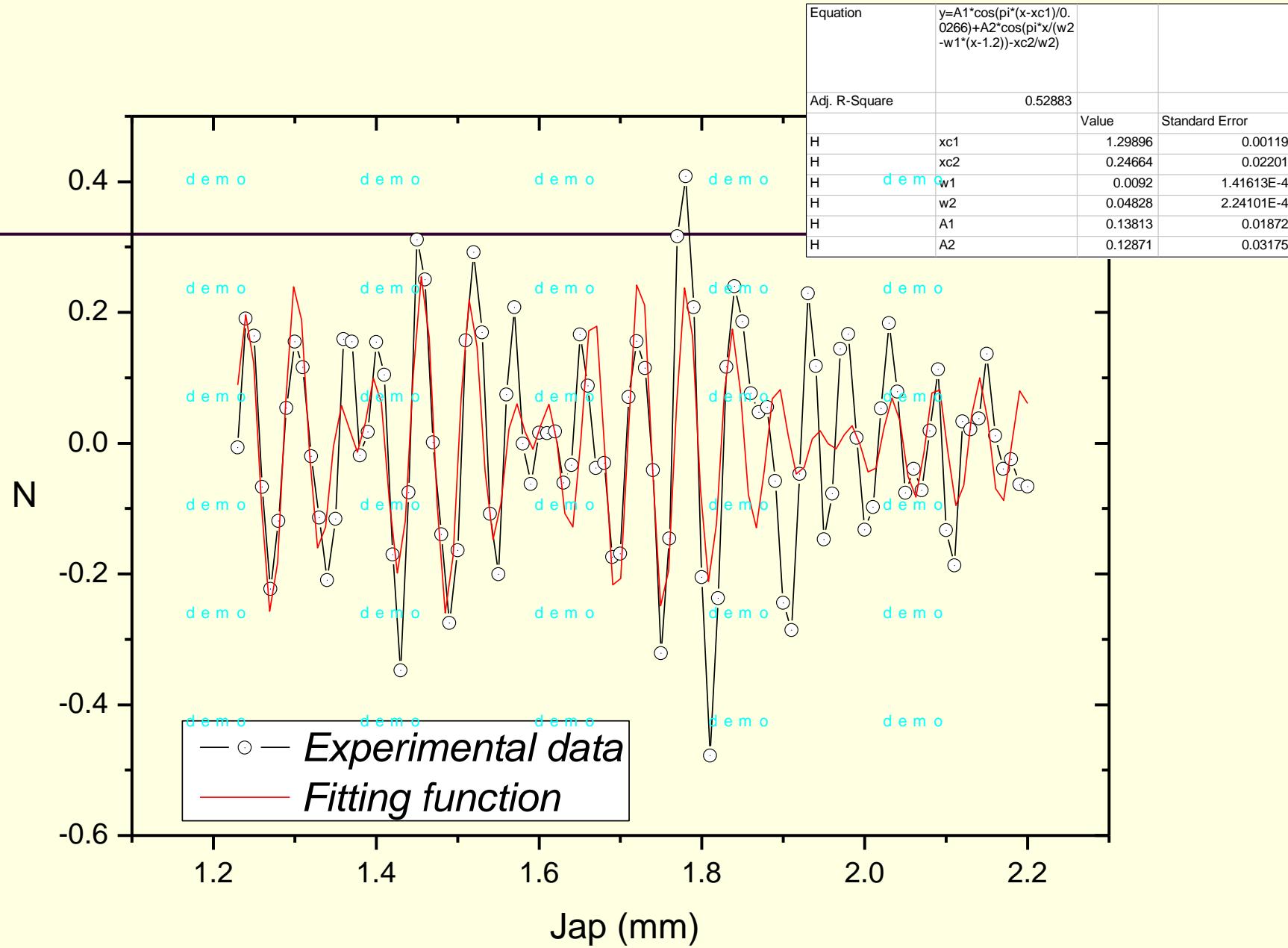
- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.



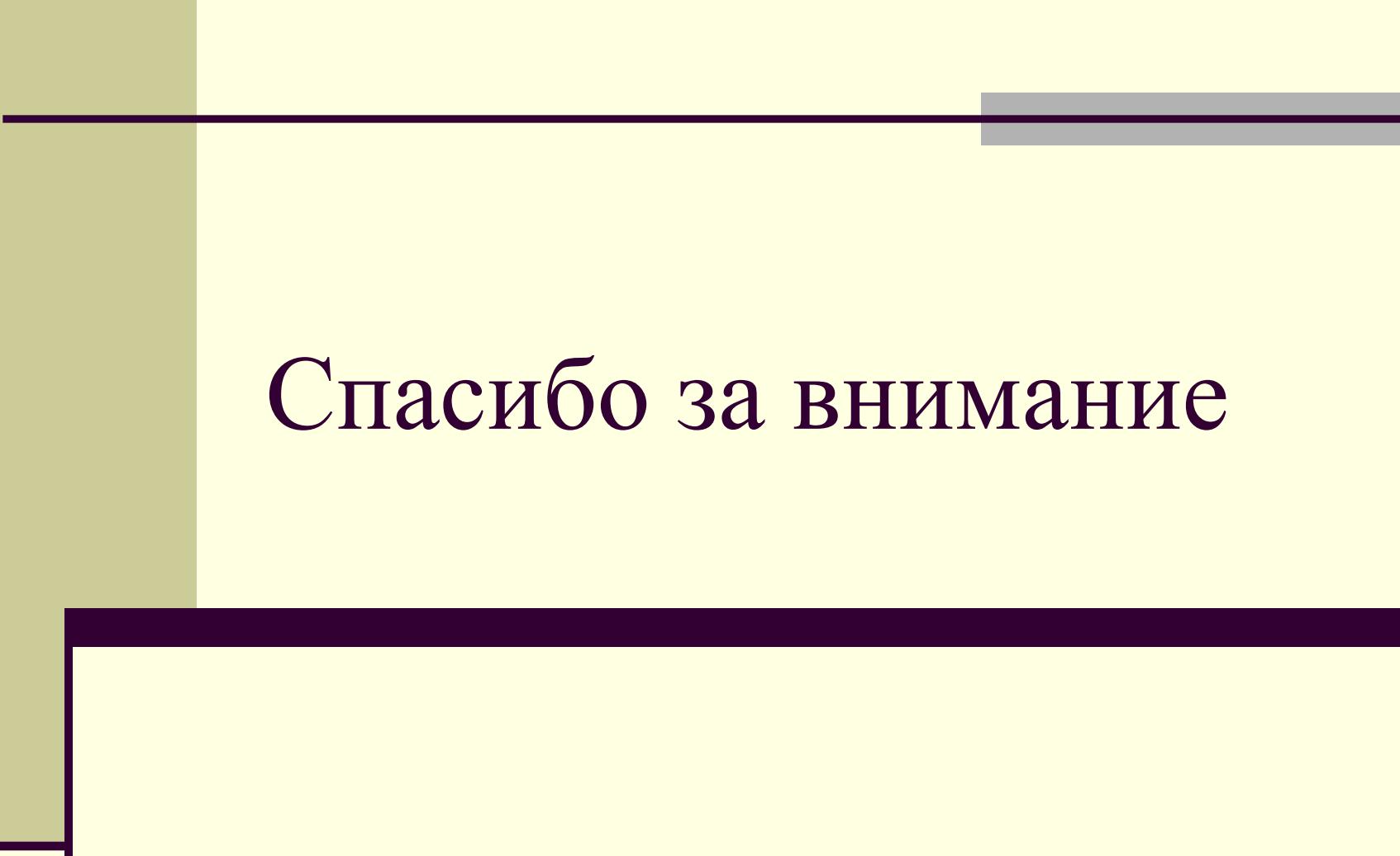
Period of second derivative corresponds to the step in the initial data

Fitting of experimental data





$$y = A_1 \cos\left(\frac{\pi(x - x_{c1})}{0.0266}\right) + A_2 \cos\left(\frac{\pi x}{(w_2 - w_1)(x - 1.2)} - \frac{x_{c2}}{w_2}\right)$$



Спасибо за внимание

