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Лекция 14. Швингеровское рассеяние. Основы нейтронной оптики.

- *Рассеяние поляризованных нейтронов на локальных электрических полях*
- *Коэффициент преломления нейтронов*
- *Магнитное зеркало*

Швингеровское рассеяние

$$\boldsymbol{\mu}_n = \mu_n \boldsymbol{\sigma} \qquad \mu_n = \gamma \mu_N < 0 \qquad \mu_N = \frac{|e|\hbar}{2m_p} \qquad \gamma = -1.91$$

$$V(\mathbf{r}) = -\boldsymbol{\mu}_n \mathbf{B}(\mathbf{r}) + O\left(m_e / m_p\right) = -\boldsymbol{\mu}_n \mathbf{B}(\mathbf{r}) + V_{SO}(\mathbf{r}) + V_F(\mathbf{r})$$

$$V_{SO}(\mathbf{r}) = \frac{\mu_n}{m_p c^2}\left(\boldsymbol{\sigma} \cdot [\mathbf{E} \times \mathbf{p}]\right)$$

$$V_F(\mathbf{r}) = \frac{\hbar \mu_n}{2m_p c^2}\left(\overline{\nabla} \cdot \mathbf{E}\right) = \frac{\hbar \mu_n}{2m_p c^2 \varepsilon_0} n(\mathbf{r}) \qquad n \rightleftarrows p + \pi^-$$

$$\left(\frac{d^2\sigma}{dE_f \, d\Omega_f}\right)_{\alpha \to \beta} = \frac{k_f}{k_i}\left(\frac{m_p}{2\pi\hbar^2}\right)^2 \left|\langle \beta f \,|V|\, \alpha i \rangle\right|^2 \delta\left(E_\alpha + E_i - E_\beta - E_f\right)$$

$$\langle f \,|V|\, i \rangle = \frac{m_p}{2\pi\hbar^2} \int d\mathbf{r} e^{-i\mathbf{k}_f \mathbf{r}} \left(V_{SO}(\mathbf{r}) + V_F(\mathbf{r})\right) e^{i\mathbf{k}_i \mathbf{r}}$$

$$\mathbf{E} = -\bar{\nabla}\phi(\mathbf{r}), \qquad \phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d\mathbf{r}'\frac{n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{(2\pi)^3\,\varepsilon_0}\int\frac{d\mathbf{q}}{q^2}n(\mathbf{q})e^{i(\mathbf{qr})}$$

$$n(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\mathbf{q}\,n(\mathbf{q})e^{i(\mathbf{qr})} \qquad \int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}e^{i(\mathbf{qr}')} = \frac{4\pi}{q^2}e^{i(\mathbf{qr})}$$

$$\mathbf{E} = -\frac{i}{(2\pi)^3\,\varepsilon_0}\int\frac{d\mathbf{q}}{q^2}n(\mathbf{q})\mathbf{q}e^{i(\mathbf{qr})} \qquad \mathbf{p} = -i\hbar\bar{\nabla}$$

$$\frac{m_p}{2\pi\hbar^2}\int d\mathbf{r}e^{-i\mathbf{k}_f\mathbf{r}}\left[\mathbf{E}\times\mathbf{p}\right]e^{i\mathbf{k}_i\mathbf{r}} = \frac{m_p}{2\pi\hbar}\int d\mathbf{r}\left[\mathbf{E}\times\mathbf{k}_i\right]e^{i\mathbf{Qr}}$$

$$= -\frac{im_p}{(2\pi)^4\,\varepsilon_0\hbar}\int d\mathbf{r}d\mathbf{q}\left[\mathbf{q}\times\mathbf{k}_i\right]\frac{n(\mathbf{q})}{q^2}e^{i(\mathbf{qr})}e^{i\mathbf{Qr}} = \frac{im_p}{2\pi\varepsilon_0\hbar}\frac{\left[\mathbf{Q}\times\mathbf{k}_i\right]}{Q^2}n(-\mathbf{Q})$$

$$= \frac{-im_p}{2\pi\varepsilon_0\hbar}\frac{\left[\mathbf{k}_f\times\mathbf{k}_i\right]}{Q^2}n(-\mathbf{Q}) = \frac{-im_p}{4\pi\varepsilon_0\hbar}n(-\mathbf{Q})\hat{\mathbf{n}}\;\mathrm{ctg}\frac{\theta}{2} \qquad Q = 2k\sin\frac{\theta}{2}$$

$$n(\mathbf{q}) = \int d\mathbf{r}\,n(\mathbf{r})e^{-i(\mathbf{qr})} \qquad n(\mathbf{q}=0) = \int d\mathbf{r}\,n(\mathbf{r}) = 0$$

$$\frac{m_p}{2\pi\hbar^2}\int d\mathbf{r}e^{-i\mathbf{k}_f\mathbf{r}}V_{SO}(\mathbf{r})e^{i\mathbf{k}_i\mathbf{r}} = \frac{\mu_n}{m_p c^2}\left(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\right)\frac{-im_p}{4\pi\varepsilon_0\hbar}n(-\mathbf{Q})\ \mathrm{ctg}\frac{\theta}{2}$$

$$= -i\gamma\frac{|e|\hbar}{2m_p}\frac{\mu_0}{4\pi\hbar}\left(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\right)n(-\mathbf{Q})\ \mathrm{ctg}\frac{\theta}{2} = -i\frac{\gamma r_p}{2|e|}\left(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\right)n(-\mathbf{Q})\ \mathrm{ctg}\frac{\theta}{2}$$

$$r_p = \frac{\mu_0}{4\pi}\frac{e^2}{m_p} \qquad \frac{r_p}{r_0} = \frac{m_e}{m_p} \approx \frac{1}{1836} \qquad \mu_0\varepsilon_0 = c^{-2}$$

$$r_0 = \frac{\mu_0}{4\pi}\frac{e^2}{m_e} = 2.818\cdot 10^{-15}\,\mathrm{m}$$ "классический радиус электрона"

$$\frac{m_p}{2\pi\hbar^2}\int d\mathbf{r}e^{-i\mathbf{k}_f\mathbf{r}}V_F(\mathbf{r})e^{i\mathbf{k}_i\mathbf{r}} = \frac{m_p}{2\pi\hbar^2}\frac{\hbar\mu_n}{2m_p c^2\varepsilon_0}n(-\mathbf{Q}) = \frac{\gamma r_p}{2|e|}n(-\mathbf{Q})$$

$$\frac{m_p}{2\pi\hbar^2}\int d\mathbf{r}e^{-i\mathbf{k}_f\mathbf{r}}\left(V_{SO}(\mathbf{r})+V_F(\mathbf{r})\right)e^{i\mathbf{k}_i\mathbf{r}} = \frac{\gamma r_p}{2|e|}n(-\mathbf{Q})\left(1-i\,\mathrm{ctg}\frac{\theta}{2}\left(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\right)\right)$$

Брэгговское рассеяние с учетом электрического поля

$$\left\langle \alpha f \,|V_{SO}(\mathbf{r}) + V_F(\mathbf{r})|\, \alpha i \right\rangle \propto \left\langle \alpha \,|\, n(-\mathbf{Q}) \,|\, \alpha \right\rangle = |e|\left(Z - f(\mathbf{Q})\right)\sum_j e^{i\mathbf{Q}\mathbf{R}_j}$$

$$f(\mathbf{Q}) = \int d\mathbf{r}\rho(\mathbf{r})e^{i\mathbf{Q}\mathbf{r}}$$

$$\hat{b} = A + B\mathbf{I}\boldsymbol{\sigma} \to A + F_E(\mathbf{Q}) + \boldsymbol{\sigma}\left(B\mathbf{I} - i\,\mathrm{ctg}\frac{\theta}{2}F_E(\mathbf{Q})\hat{\mathbf{n}}\right)$$

$$F_E(\mathbf{Q}) = \frac{\gamma r_p}{2}\left(Z - f(\mathbf{Q})\right)$$

$$F_E(\mathbf{Q} = 0) = 0$$

$$\overline{\left\langle \uparrow \middle| \hat{b} \middle| \uparrow \right\rangle} = A + F_E(\mathbf{Q}) - i\,\mathrm{ctg}\frac{\theta}{2}F_E(\mathbf{Q})\hat{n}_z$$

$$\overline{\left\langle \downarrow \middle| \hat{b} \middle| \downarrow \right\rangle} = A + F_E(\mathbf{Q}) + i\,\mathrm{ctg}\frac{\theta}{2}F_E(\mathbf{Q})\hat{n}_z$$

$$\overline{\left\langle \uparrow \middle| \hat{b} \middle| \downarrow \right\rangle} = -i\,\mathrm{ctg}\frac{\theta}{2}F_E(\mathbf{Q})\left(\hat{n}_x + i\hat{n}_y\right)$$

$$\overline{\left\langle \downarrow \middle| \hat{b} \middle| \uparrow \right\rangle} = -i\,\mathrm{ctg}\frac{\theta}{2}F_E(\mathbf{Q})\left(\hat{n}_x - i\hat{n}_y\right)$$

Брэгговское рассеяние с учетом электрического поля

$$\left(\frac{d\sigma}{d\Omega_f}\right)_{\mathrm{coh,el}} = N\frac{(2\pi)^3}{\mathrm{v}_0}\sum_{\boldsymbol{\tau}}\delta(\mathbf{Q}-\boldsymbol{\tau})$$

$$\times\left(f\left(\left|\overline{\langle\uparrow|\hat{b}|\uparrow\rangle}\right|^2+\left|\overline{\langle\uparrow|\hat{b}|\downarrow\rangle}\right|^2\right)+(1-f)\left(\left|\overline{\langle\downarrow|\hat{b}|\downarrow\rangle}\right|^2+\left|\overline{\langle\downarrow|\hat{b}|\uparrow\rangle}\right|^2\right)\right)$$

$$P = f-(1-f)=2f-1$$

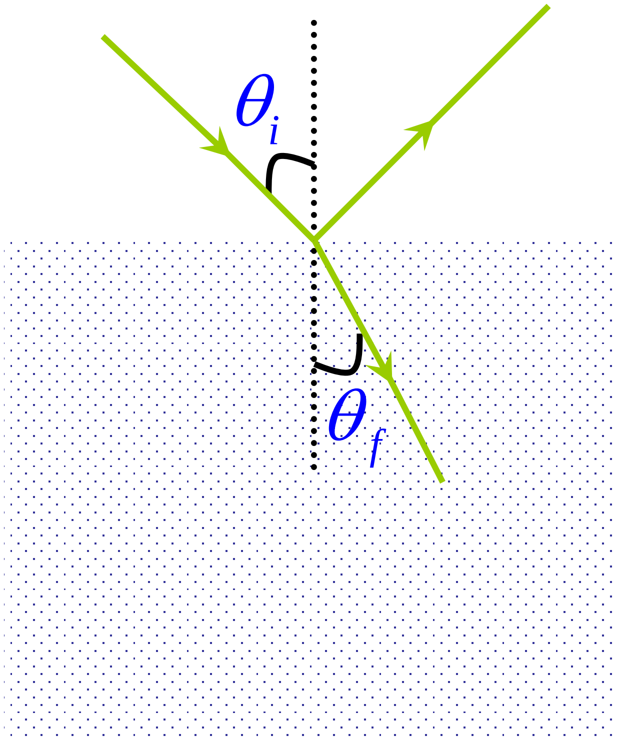
$$\boxed{\begin{aligned}\left(\frac{d\sigma}{d\Omega_f}\right)_{\mathrm{coh,el}} &= N\frac{(2\pi)^3}{\mathrm{v}_0}\sum_{\boldsymbol{\tau}}\delta(\mathbf{Q}-\boldsymbol{\tau})\Bigg(A^2+\left|F_E(\mathbf{Q})\right|^2+\mathrm{ctg}^2\frac{\theta}{2}\left|F_E(\mathbf{Q})\right|^2\\ &+2\,\mathrm{Re}\left(AF_E(\mathbf{Q})\right)+2\,\mathrm{ctg}\frac{\theta}{2}\,\mathrm{Im}\left(AF_E(\mathbf{Q})\right)\left(\mathbf{P}\cdot\hat{\mathbf{n}}\right)\Bigg)\end{aligned}}$$

Коэффициент преломления нейтронов

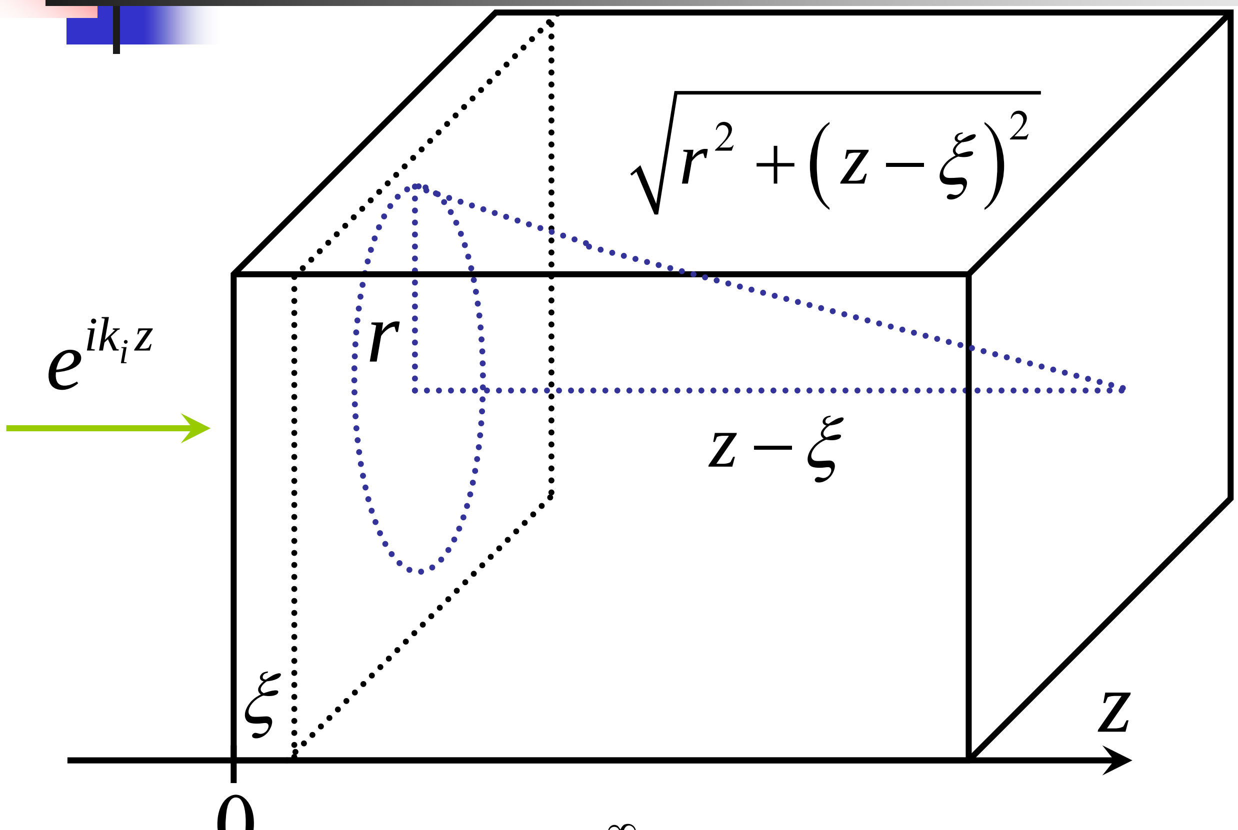
Оптические эффекты – результат интерференции падающих и рассеянных нейтронов

$$n = \frac{k_f}{k_i} = \frac{\sin\theta_i}{\sin\theta_f}$$

$$\sin\theta_c = n$$



Коэффициент преломления нейтронов



$$\Phi_{\Delta z} = e^{ik_i z} - \int_0^\infty \sum_j \overline{b_j} \rho_j 2\pi r dr \Delta z e^{ik_i \xi} \frac{e^{ik_i \sqrt{r^2 + (z-\xi)^2}}}{\sqrt{r^2 + (z-\xi)^2}}$$

Коэффициент преломления нейтронов

$$\int_0^\infty \sum_j \overline{b_j}\rho_j 2\pi r dr \Delta z e^{ik_i\xi} \frac{e^{ik_i\sqrt{r^2+(z-\xi)^2}}}{\sqrt{r^2+(z-\xi)^2}}$$

$$x = \sqrt{r^2+(z-\xi)^2} \qquad dx = \frac{rdr}{\sqrt{r^2+(z-\xi)^2}} = \frac{rdr}{x}$$

$$= 2\pi\Delta z e^{ik_i\xi} \sum_j \overline{b_j}\rho_j \int_{|z-\xi|}^{\infty} x dx \frac{e^{ik_i x}}{x}$$

$$= 2\pi\Delta z e^{ik_i\xi} \sum_j \overline{b_j}\rho_j \int_{|z-\xi|}^{\infty} e^{ik_i x} dx = -2\pi\Delta z e^{ik_i\xi} \sum_j \overline{b_j}\rho_j \frac{e^{ik_i|z-\xi|}}{ik_i}$$

Коэффициент преломления нейтронов

$\underline{z > \xi}$

$$\Phi_{\Delta z} = e^{ik_i z} + 2\pi\Delta z e^{ik_i\xi}\sum_j \overline{b_j}\rho_j \frac{e^{ik_i(z-\xi)}}{ik_i} = e^{ik_i z}\left(1 - i\frac{2\pi}{k}\Delta z\sum_j \overline{b_j}\rho_j\right)$$

$$\Phi = e^{ik_i z}\left(1 - i\frac{2\pi}{k_i}\Delta z\sum_j \overline{b_j}\rho_j\right)^{\frac{z}{\Delta z}} \approx e^{ik_i z}\left(e^{-i\frac{2\pi}{k_i}\Delta z\sum_j \overline{b_j}\rho_j}\right)^{\frac{z}{\Delta z}}$$

$$= e^{ik_i z}e^{-i\frac{2\pi}{k_i}z\sum_j \overline{b_j}\rho_j} = e^{ik_f z}$$

$$k_f = k_i\left(1 - \frac{2\pi}{k_i^2}\sum_j \overline{b_j}\rho_j\right) \quad \Rightarrow \quad \boxed{n = 1 - \frac{2\pi}{k_i^2}\sum_j \overline{b_j}\rho_j = 1 - \frac{\lambda^2}{2\pi}\sum_j \overline{b_j}\rho_j}$$

Коэффициент преломления нейтронов

$$n = 1 - \frac{\lambda^2}{2\pi}\sum_j \overline{b_j}\rho_j$$

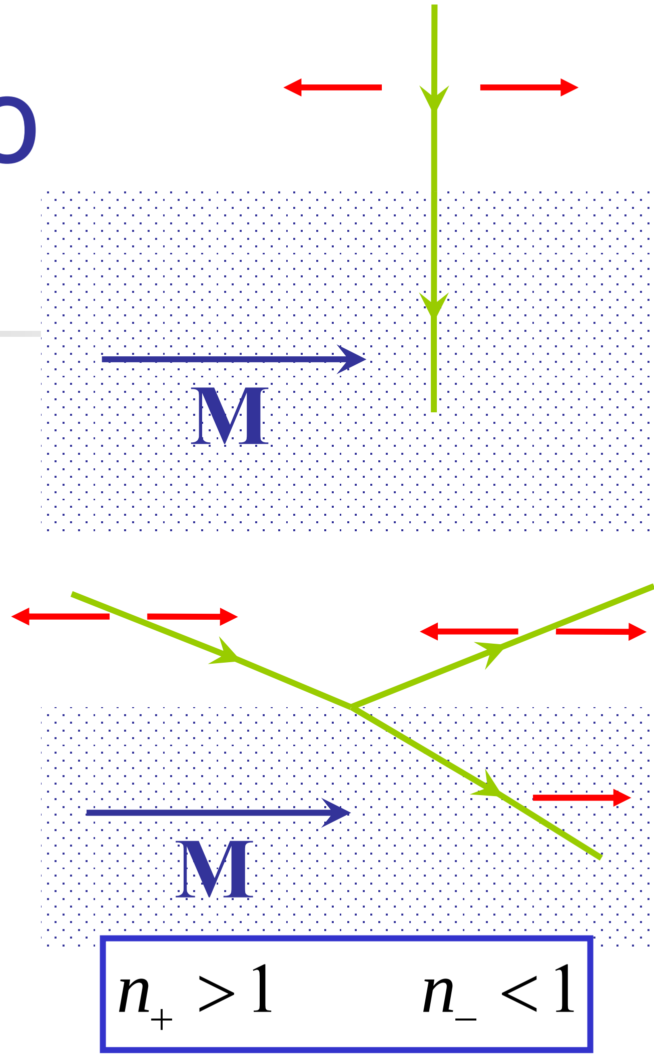
$$\frac{\lambda^2}{2\pi}\sum_j \overline{b_j}\rho_j \sim \frac{a^2}{2\pi}10^{-14}\frac{1}{a^3} = \frac{10^{-14}}{10^{-10}2\pi} \sim 10^{-5}$$

Ni

λ, $\mathring{A}$	1 – n	$\frac{\pi}{2}-\theta_c$
1	1.5 10^{-6}	6′
5	3.7 10^{-5}	30′

Z	Nucleus	b (10^{-15} m
1	^{1}H	-3.742
1	^{2}D	6.674
2	^{3}He	5.74
2	^{4}He	3.26
3	Li	-1.90
4	Be	7.79
5	B	5.30
6	C	6.6484
7	N	9.36
8	O	5.805
9	F	5.654
10	Ne	4.566
11	Na	3.63
12	Mg	5.375
13	Al	3.449
14	Si	4.1507
15	P	5.13
16	S	2.847

Магнитное зеркало



$$\mathbf{C}_j = \gamma r_0 F(\mathbf{Q})\left[\hat{\mathbf{Q}} \times \left[\mathbf{S}_j \times \hat{\mathbf{Q}}\right]\right] = \gamma r_0 F(\mathbf{Q})\mathbf{S}_j$$

$$T_j = A_j + \boldsymbol{\sigma}\left(B_j \mathbf{I}_j - \mathbf{C}_j\right)$$

$$\left\langle \uparrow \middle| T_j \middle| \uparrow \right\rangle = \bar{b} - C_j^z = \bar{b} - \gamma r_0 \left\langle S^z \right\rangle$$

$$\left\langle \downarrow \middle| T_j \middle| \downarrow \right\rangle = \bar{b} + C_j^z = \bar{b} + \gamma r_0 \left\langle S^z \right\rangle$$

$$\left\langle \uparrow \middle| T_j \middle| \downarrow \right\rangle = -\left(C_j^x - iC_j^y\right) = 0$$

$$\left\langle \downarrow \middle| T_j \middle| \uparrow \right\rangle = -\left(C_j^x + iC_j^y\right) = 0$$

$$\boxed{n_+ = 1 - \frac{\lambda^2}{2\pi}\sum_j \left(\overline{b_j} - \gamma r_0 \left\langle S^z \right\rangle\right)\rho_j \qquad n_- = 1 - \frac{\lambda^2}{2\pi}\sum_j \left(\overline{b_j} + \gamma r_0 \left\langle S^z \right\rangle\right)\rho_j}$$